A MATHEMATICAL MODEL FOR MULTI-PERIOD SURGICAL SCHEDULING WITH CAPACITY CONSTRAINT

W. Wisittipanich[1], C. Boonmee[1], K. Khwanngern[2], W. Chattinnawat[1] and M. Woschank[3]

[1]Faculty of Engineering,
Chiang Mai University, 50200
Chiang Mai, Thailand.

[2]Faculty of Medicine,
Chiang Mai University, 50200
Chiang Mai, Thailand.

[3]Department Economic and Business Management,
Montanuniversitaet Leoben, 8700
Leoben, Austria.

Corresponding Author's Email: [1]warisa.o@gmail.com

Article History: Received 15 December 2020; Revised 30 May 2021; Accepted 18 August 2021

ABSTRACT: Surgical scheduling is a decision-making process that plays a crucial role in medical treatment. This research aimed to employ a mathematical programming model to solve the surgical scheduling problem. A mathematical model for a multi-period surgical scheduling problem with capacity constraints over a particular time horizon was proposed in this study. The goal was to schedule a list of patients who must undergo various kinds of operations. In particular, each operation of a patient must be performed during a specific time period by one of the eligible hospitals. In addition, each hospital has limited surgical capacity for each time period. The surgical scheduling problem was formulated with a multi-objective model using the weighted sum approach of two objectives: minimization of makespan and minimization of the total least preference assignment score. The experiment was executed using the simulated data according to the real treatments of cleft lip and cleft palate patients. The numerical results showed that the model yielded the optimal schedule which satisfied all constraints. The solution obtained from the model was similar to the current method. The proposed model also showed its superiority in terms of computational time and will be further used as a smart decision tool in the hospital scheduling.

KEYWORDS: *Mathematical Model; Surgical Scheduling; Multi-Period; Capacity Constraint; Scheduling Problem*

1.0 INTRODUCTION

One of the most important roles regarding the managerial aspect in health care service is providing prompt health services to patients [1]. The surgical scheduling problem is a complex problem for many operating room administrators. In general, this problem consists of selecting surgeries to operating rooms or hospitals, specifying the surgical period, and identifying the required resources [2]. Therefore, surgical scheduling is a solution to support a decision-maker when hospital management has certain objectives for improving their current systems; for example, improving access, enhancing quality, and reducing the cost of the health care system or maximize the level of patient satisfaction. However, a tradeoff between conflict objectives could occur. Increasing patient flow and reducing lead time would be a benefit for patients but these improvements can lead to an increase in costs [3]. Recently, Hamid et al. [4] proposed a multi-objective mathematical model for scheduling inpatient surgeries. The model included three objective functions; minimizing total costs related to surgeries, maximizing the level of patient satisfaction according to their priorities, and maximizing the compatibility among the surgical team members.

Typically, a hospital administrator schedules surgical operations based on the availability of operating surgeons and operating room (OR) and suitable timing of the patient [5]. Designing a surgical scheduling system is an important task to enhance healthcare service and ensure optimal utilization of the costly medical resources while assuring patient satisfaction. Recently, the number of research on patient scheduling has been increased to maximize patient satisfaction. Donahue et al. [6] claimed that many aspects of the patient experience can affect patient satisfaction, for instance, time spent with the provider, continuity of care, adequate access to primary care or a strong patient-provider relationship.

The surgical scheduling problem can be classified based on the planning horizon. Long-term planning aims to create and/or upgrade facilities, medium-term planning tries to allocate surgical time periods to surgeons, and short-term planning aims to allocate patients to days and times within time periods. At the last minute, some adjustments can be made at short-term planning before the schedule is executed. Capacity planning is a challenging issue of dealing with various components, for example, short product life cycle, high volume, product varieties or long processing time [7]. In surgical scheduling problems, capacity planning composed of three components. The first component concerns physical aspects such as the number of rooms and equipment. The second component involves human resources, for example, the number, and type of surgical practices, anesthesiologists, and other OR staff available. The third component is resource availability which includes the number of hours that ORs open and how those hours are parceled out [8]. Numerous literature has been studied on operational capacity planning in hospitals. One unit that is of particular interest is the operating room (OR) [9]. In most cases, mixed-integer programming (MIP) was used to schedule elective patients for each day of the week from different categories to be admitted into the hospital subjected to scarce resources such as beds and operating rooms [10]. Apart from capacity planning mentioned previously, Silva and De Souza [2] and Min and Yih [11] solved surgical scheduling problems under a stochastic environment based on uncertainty parameters, for example, surgery durations, the arrival of emergency surgeries, and capacity of the surgical intensive care unit.

This research aims to employ a mathematical programming model to solve the surgical scheduling problem at the operational planning level so that operations of patients can be planned. This study presents a novel mathematical model for a multi-period surgical scheduling problem with capacity constraints over a particular time horizon. The problem was formulated with a multi-objective model using the weighted sum approach of two objectives: minimization of makespan and minimization of total least preference assignment score. Next, the computational experiments are executed using LINGO optimization solver for model analysis. Finally, the discussion and conclusion are provided.

2.0 MODEL FORMULATION

As mentioned earlier, the surgical scheduling problem in this study aims to schedule all patients who must undergo several surgical requirements in a particular time horizon. Different operations of each patient can be performed at any hospital selected from a set of eligible hospitals. In addition, for each time period, different hospitals have different limited capacities to perform surgery. Therefore, the surgical schedule is planned for each hospital on multiple periods over a particular time horizon. In this section, a mixed-integer programming (MIP) model is proposed to represent the surgical scheduling problem. The decisions in this problem include such as i) identifying a list of selected surgeries or operations, ii) assigning all operations of patients to the selected hospital, and iii) identifying the time period to perform selected surgeries. The model consists of two objectives which are minimization of makespan and minimization of the total least preference assignment score which is based on patient location. This model used a weighted sum approach to combine two objectives into a single objective.

The main assumptions of the model were summarized as follows:

i. Each patient has different symptoms. Consequently, each patient requires different operational requirements.
ii. Some operations can be performed at certain hospitals only.
iii. The treatment time of each operation is defined as a deterministic variable.
iv. The transferring time of a patient between hospitals is not considered.
v. There are no preemptions in scheduling.
vi. All patients have equal priority.

In order to formulate a mathematical model for a multi-objective surgical scheduling problem, the notations for indices, parameters, and decision variables are defined as follows:

The indicators are such as

i = Patient (i = 1,… ,n); j = Treatment or Operation (j = 1,…, q); k = Hospital (k = 1,…, m); t = Time period (t = 1,…, u).

The decision variable represents such as

X_{ijkt} 1 if patient i is assigned to hospital k for operation j in period t, or 0 otherwise.
Ck_{ij} Treatment completion time of patient i operation j.
Cx_{ijk} Treatment completion time of patient i operation j at hospital k.
Cmax Maximum completion time of all patients.
sc_{ijkt} Assignment score of patient i operation j to hospital k in period t.
Tsc Total least preference score from assignment.

Parameters depict as

p_{ijkt} Processing time for treatment patient i operation j in hospital k in period t.
r_{ij} Ready time for starting operation j of patient i.
d_{ij} Maximum date for completing operation j of patient i.

$capa_{jkt}$	Maximum capacity of hospital k that can treat operation j in period t.
w_{ik}	Least preference score of patient i being treated at hospital k
tp_t	Time period t.
e_{ijkt}	Hospital eligibility restrictions, $e_{ijkt} = 1$ if hospital k can treat patient i operation j in period t, or 0 otherwise.
a_{ij}	Treatment requirement, $a_{ij} = 1$ if patient i need to treat an operation j, or 0 otherwise.

Equation (1) shows an objective function of this model. Two objectives of makespan and total least preference score are given the same weight and combined into a single objective.

$$\text{Minimize} Z = \left(0.5 * C\max\right) + \left(0.5 * Tsc\right) \tag{1}$$

Equation (2) ensures the patient must be treated for one operation by only one hospital on one time period ($a_{ij} = 1$) and there is no need to assign a patient to any hospital at any time period if patient i are not required for an operation j ($a_{ij} = 0$).

$$\sum_{t=1}^{u} \sum_{k=1}^{m} X_{ijkt} = a_{ij}; \forall_{i,j} \tag{2}$$

Equation (3) specifies each patient can be assigned to any eligible hospital to treat the required operation.

$$X_{ijkt} \le e_{ijkt}; \forall_{i,j,k,t} \tag{3}$$

Equation (4) is a precedence constraint to ensure the completion time of an operation must be greater than or equal to its ready time plus its processing time.

$$C_{ijk} - C_{ij-1k} \ge \sum_{k=1}^{m} \left(p_{ijkt} * X_{ijkt}\right); \forall_{i,j,k,t} \ne 1 \tag{4}$$

Equations (5) and (6) state the completion time of patient i with operation j at hospital k must occur between patient's ready time plus processing time of the operation and the maximum specified age of a patient to perform the operation.

$$C_{ijk} \ge r_{ij} + p_{ijkt}; \forall_{i,j,k,t} \tag{5}$$

$$C_{ijk} \le d_{ij}; \forall_{i,j,k,t} \tag{6}$$

Equations (7) and (8) define the completion time of patient i with operation j at hospital k.

$$Max\left(Cx_{ijk}\right) = Ck_{ij}; \forall_{i,j,k} \tag{7}$$

$$Ck_{ij} * a_{ij} = \sum_{t=1}^{u}\sum_{k=1}^{m} X_{ijkt} * tp_{t}; \forall_{i,j} \tag{8}$$

Equation (9) expresses a capacity constraint of hospital k that can treat operation j in time period t.

$$\sum_{i=1}^{n}\sum_{j=1}^{1} X_{ijkt} \le capa_{jkt}; \forall_{j,k,t} \tag{9}$$

Equation (10) calculates the assignment score for patient i that is being treated operation j in hospital k, while Equation (11) calculates total assignment score of all patients.

$$sc_{ijkt} = X_{ijkt} * w_{ik}; \forall_{i,j,k,t} \tag{10}$$

$$Tsc = \sum_{i=1}^{n}\sum_{j=1}^{1}\sum_{k=1}^{m}\sum_{t=1}^{u} sc_{ijkt} \tag{11}$$

Equation (12) specifies the makespan, Cmax (maximum completion time of all patients).

$$C\max = Max\left(C_{ijk}\right); \forall i, j, k; a_{ij} = 1 \tag{12}$$

Equation (13) specifies the decision variable X_{ijkt} is binary.

$$X_{ijkt} \in \left(0,1\right) \tag{13}$$

3.0 RESULTS AND DISCUSSION

In this study, a numerical example is given to illustrate solution methods of the proposed surgical scheduling model. In order to make the problem practical, the computational experiments are executed using the simulated data based on the real treatment data of cleft lip and cleft palate patients. Three main parameters, the number of patients, list of operations, and the number of hospitals are generated along with other parameters such as hospital eligibility for all operations, hospital capacity, and hospital preference for patients. Each operation is scheduled weekly over a planning horizon. An example of a surgical scheduling problem with 16 patients and 3 hospitals is used for model analysis. Each patient can be treated for up to four maximum operations. The planning period or time horizon is set as 28 weeks. Table 1 shows the surgical capacity of three hospitals. In detail, each hospital has limited surgical capacity for each time period. For example, the maximum surgical capacity of Hospital 1 is 5 times per week for the 1st operation.

Table 1: Surgical capacity for different hospitals

Operation no.	Surgical capacity per week		
	Hospital 1	Hospital 2	Hospital 3
1st	5	5	0
2nd	0	0	5
3rd	5	0	5
4th	2	2	0

The problem is solved by an exact method using LINGO optimization solver version 14.0. Figure 1 shows the optimal schedule of operations for all patients with assigned hospitals. According to the result obtained from LINGO optimization solver, the maximum completion time of all patients is 21 weeks and the total least preference score is 81.

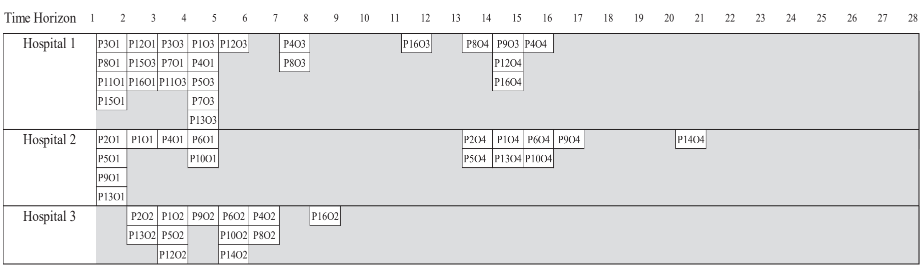

Figure 1: An optimal solution of the surgical scheduling problem

The solution obtained from LINGO optimization solver is also validated to ensure the correctness of the proposed model. Table 2 shows the validation result of an optimal solution with correct assignment and operation restrictions.

Table 2: Model validation results

Constraints	Results
Assignment - Patient and operation assignment - Hospital assignment	**Correct assignment:** - Each operation of a patient was treated by only one hospital. - Each patient was assigned to one of the eligible hospitals to treat a requited operation.
Sequence - Patient age - Operation sequence	**Correct sequence:** - Each operation was performed between the patient's ready time and the maximum specified age. - All operations are scheduled in the correct sequence.
Hospital Capacity	- For all hospitals, the number of assigned operations for each period is lower than or equal to their maximum capacity.

According to validation results from Table 2, the obtained optimal schedule satisfies all constraints; assignment constraints, sequence constraints, and hospital capacity constraint. Thus, the proposed model can be used to solve the surgical scheduling problem under this study's circumstances. In addition, the optimal schedule obtained from the model is compared with a current solution from the Craniofacial Center of Chiang Mai University, Thailand. It is found that the obtained schedule is similar to the solution from the current practice which was done manually.

However, it is observed that, for such a small case, the current practice which manually schedules operations of all patients took about 3 weeks to find a complete solution, while the proposed method only took 35 seconds to obtain an optimal solution. For these reasons, it can be concluded that the proposed method significantly outperformed the current method in terms of computational time.

4.0 CONCLUSION

In this study, the multi-period surgical scheduling problem with limited surgical capacity was proposed. The problem was formulated as a mixed-integer programming (MIP) model. The aim was to schedule a list of patients who must undergo various kinds of operations by a set of eligible hospitals. Two objectives were considered in the proposed model; minimization of makespan and minimization of total least preference score. The weighted sum approach was used to combine two objectives in a single objective.

Then, an instance inspired by real treatments of cleft lip and cleft palate patients was generated to conduct a computational experiment for model analysis. The result showed that the model yields correct assignment and operation sequence respected to all constraints. The solution obtained from the proposed method was similar to that from the current manual method. However, the proposed method significantly outperformed the current method in terms of computational time. Therefore, the proposed mathematical programming model has the potential to bring significant improvements to real-world practice. In this study, the surgical scheduling problem was discussed under the assumption that the surgery durations and capacity are deterministic variables, and only non-preemptive cases are considered.

However, in practice, some of these assumptions can be unrealistic. For example, a hospital may be subjected to unpredictable conditions of their surgical capacity and surgical durations. Furthermore, the arrival of emergency or urgent surgeries may occur and result in preemptions in the scheduling. Hence, further research should focus on stochastic modeling for handling the uncertainty in real-world practices.

ACKNOWLEDGMENTS

This paper was supported by Research Assistant Scholarship Project, Faculty of Engineering, Chiang Mai University, Thailand.

REFERENCES

[1] B. Cardoen, E. Demeulemeester, and J. Beliën, "Operating room planning and scheduling: A literature review", *European Journal of Operational Research*, vol. 201, no. 3, pp. 921-932, 2010.

[2] T. A. Silva and M. C. De Souza, "Surgical scheduling under uncertainty by approximate dynamic programming", *Omega*, vol. 95, pp. 1-15, 2020.

[3] J. Drupsteen, T. V. D. Vaart, and D. P. V. Donk, "Integrative practices in hospitals and their impact on patient flow", *International Journal of Operations & Production Management*, vol. 33, no. 7, pp. 912-933, 2013.

[4] M. Hamid, M. M. Nasiri, F. Werner, F. Sheikhahmadi, and M. Zhalechian, "Operating room scheduling by considering the decision-making styles of surgical team members: a comprehensive approach", *Computers & Operations Research*, vol. 108, pp. 166-181, 2019.

[5] E. J. Langer, "The psychology of chance". *Journal for the Theory of Social Behaviour*, vol. 7, no. 2, pp. 185-203, 1977.

[6] R. Donahue, D. Russell, C. de Riese, C. Smith, W. T. de Riese, and A. Medway, "Patients willing to wait: arrival time, wait time and patient satisfaction in an ambulatory urology clinic", *Urology Practice*, vol. 4, no. 1, pp. 1-6, 2017.

[7] H. M. Asih, K. E. Chong, and M. Faishal, "Capacity planning and product allocations under testing time uncertainty in electronic industry", *Journal of Advanced Manufacturing Technology*, vol. 12, no. 1, pp. 103-116, 2018.

[8] J. H. May, W. E. Spangler, D. P. Strum, and L. G. Vargas, "The surgical scheduling problem: Current research and future opportunities", *Production and Operations Management*, vol. 20, no. 3, pp. 392-405, 2011.

[9] D. Gartner and R. Kolisch, "Scheduling the hospital-wide flow of elective patients", *European Journal of Operational Research*, vol. 233, no. 3, pp. 689-699, 2014.

[10] J. M. H. Vissers, I. J. Adan, and J. A. Bekkers, "Patient mix optimization in tactical cardiothoracic surgery planning: a case study", *IMA Journal of Management Mathematics*, vol. 16, no. 3, pp. 281-304, 2005.

[11] D. Min and Y. Yih, "An elective surgery scheduling problem considering patient priority", *Computers & Operations Research*, vol. 37, no. 6, pp. 1091-1099, 2010.