CLASSICAL LAMINATES THEORY: APPLICATION TO COMBINED COMPOSITE CFRP/GFRP

N.A.A. Md Zahir[1], M.A.M. Daud[1], A.F. Ab Ghani[2] and F. Alami[3]

[1]Faculty of Mechanical Engineering,
Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100 Durian
Tunggal, Melaka, Malaysia.

[2]Faculty of Mechanical and Manufacturing Engineering Technology,
Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100 Durian
Tunggal, Melaka, Malaysia.

[3]Department of Civil Engineering,
University of Lampung,
35145 Lampung, Indonesia.

Corresponding Author's Email: [1]aizatul_ain92@yahoo.com

Article History: Received 22 September 2018; Revised 20 July 2019;
Accepted 13 October 2020

ABSTRACT: The advantages of using hybrid composite in terms of mechanical properties and flexibility of choosing materials made them extensively used in high precision industries. However, the study involving hybrid composite for optimized design is complicated and involves complex mathematical computation. In order to unlock the full potential of hybrid composite, the number of parameters which influence the modulus of elasticity of hybrid composite such as lay-up sequence, configurations and thickness layup is studied for the purpose of design optimization for hybrid composite CFRP/GFRP laminates. In this research, Glass Fiber Reinforced Polymer (GFRP) ply is added into high modulus of Carbon Fiber Reinforced Polymer (CFRP). The method utilized in this research is analytical approach via MATLAB R2012b along with validation by using eLamX2 composite laminates software. The outcomes of this study indicate that the higher number of CFRP plies in the hybrid composite laminates will increase the modulus of elasticity of the laminates. The position of CFRP ply at the outermost of hybrid composite laminates increased the bending stiffness of overall laminates. For CFRP reduced thickness, the extensional stiffness decreases when compared to baseline and third method. Thinner GFRP layup may increase the modulus of elasticity of composite laminates.

KEYWORDS: *Hybrid Composite; CFRP; GFRP; Design Optimization; MATLAB*

1.0 INTRODUCTION

Composites material have been known for their ability to replace the conventional material because of their higher strength-to-weight ratio, tailored design, corrosion resistant, durable and reduced long term maintenance. Although replacing metal to fully carbon fiber may be advantageous in higher strength-to-weight ratio, however this method will use a lot of money since the cost to produce carbon fiber is high. Thus, another method that can be considered is by producing a Hybrid Fiber Reinforced Polymer (HFRP) by adding Glass Fiber Reinforced Polymer (GFRP) into high modulus unidirectional Carbon Fiber Reinforced Polymer (CFRP). Hybrid composite differs from conventional composites in terms of having a combination of two or more reinforcement fibers in a single matrix or single reinforcing fiber in multiple matrix or multiple reinforcing fibers in multiple matrix which offers better flexibility than the conventional composites [1]. In terms of hardness, the contributing factors to the composites is adding the maleic anhydrite and fiber composition inside the composite [2]. The components of high modulus fiber, CFRP functions to provide stiffness and load bearing qualities, whereas the low modulus fiber, GFRP makes the hybrid composite becomes more tolerant to damage and at the same time reduce the material cost of hybrid composites.

The continuous application of hybrid composite with infancy level of knowledge have become a major concern for the researchers to dig deep in order to enhance the knowledge of properties and behavior of these materials [3, 5-6]. Another aspect that has not been thoroughly explored is the optimum thickness for more efficient mechanical properties of hybrid composite material. A number of research studies have focused on combination of CFRP and GFRP to investigate the strength and stiffness enhancement since the combination of carbon and glass is the common reinforcement fibers to produce hybrid composite. A comparative study of tensile and compression properties of uni-directional (UD) hybrid composites carbon and glass with non-hybrid composite was conducted [4]. Fabrics and filament wound fiber configurations of hybrid composites behave similarly in tensile loading whereas in compression loading filament wound laminates performed better than hybrid fabrics. The developments of failure in UD hybrid composites was predicted by using fiber break models [5]. By increasing the number of carbon plies in the laminates, the hybrid effect decreases which imply that the failure strain can be applied as reference failure strain for calculating hybrid effect.

A tensile and compressive test were conducted in order to determine the effective material properties of HFRP composites by using full scale FRP I-beam [6]. All beams failed in a brittle behavior with a delamination at interfacial layers and load carrying capacity of beams is dependent on the content of CFRP and GFRP. Flexural stiffness and strength enhancement of horizontally glued laminated wood beams with GFRP and CFRP composite sheets were determined [7]. This study illustrates that in order to strengthen laminated wood beams, FRP proved to be efficient reinforcement material. Flexural strengthening of concrete beams using CFRP, GFRP and hybrid FRP sheets is studied [8]. GFRP was recognized to be a good material for strengthening because of its high deformability, good impact and break resistance properties compensate with its lower elastic modulus than carbon fiber. Effect of variation thickness on tensile properties of hybrid polymer composite (glass fiber-carbon fiber- graphite) and GFRP composite was made [9]. The comparison hybrid and GFRP at different thickness shown that the tensile strengths of hybrid and GFRP composites of 4 mm thickness is less when compared to the difference of strengths of 2 mm and 3 mm thick composites suggest that weak bond of 4 mm thick hybrid composite lamina. The effect of FRP thickness on energy absorption of metal-FRP square tubes subjected to axial compressive loading was determined [10]. It was proved that the thickness of composite section plays an important parameter that can change the crushing mode of stainless steel tubes and also, in some cases, lower its energy absorption due to change in plastic deformation. The solution obtained from the study of composite structure design for strength and stiffness with respect to ply thickness based on the sequential convex programming is not global optimum due to lack of convexity resulting from the orientation/thickness formulation [11].

In order to enhance the knowledge related to hybrid composite performance, standard epoxy matrix material with the reinforcement fibers of UD carbon and glass were chosen to perform the non-destructive evaluation via analytical approach. The composition of CFRP and GFRP was varied along with the position in the laminates; also the variation of thickness was studied to investigate the mechanical properties behavior.

The comparison is highlighted in terms of extensional and bending stiffness in longitudinal direction (1-direction) and the result achieved is presented.

2.0 METHODOLOGY

2.1 Materials

Material used in this recent study was commercially available carbon/glass fiber reinforced polymer from Cytec. The overall fiber volume fraction of the composite was approximately 60%. The basic concept for underlying the fiber reinforced composite material are stiffness and strength, thus, the output that was concerned in this study was elastic modulus in 1-direction, E11 since it is an important parameter in longitudinal direction. The mechanical properties of CFRP/GFRP is in Table 1 as follows:

Table 1: Material database for CFRP and GFRP

Properties	CFRP	GFRP
Longitudinal elastic modulus, E1 (GPa)	128.80	45.20
Transverse elastic modulus, E2 (GPa)	9.30	14.10
Major Poisson's ratio, V12	0.34	0.29
Shear Modulus, G12 (GPa)	3.37	6.30
Thickness per layer (mm)	0.45	0.14

2.2 Stacking Sequences and Configurations

The UD CFRP and GFRP layup was arranged into nine different configurations with different composition of carbon and glass as shown in Table 2 below. The red region indicates CFRP layup while yellow region indicates GFRP layup. The factors that have been taken considered to develop these different configurations are; the position of CFRP and GFRP, the number of GFRP layer incorporated into CFRP laminates and total thickness of composite laminates.

Table 2: Nine stacking configurations of CFRP/GFRP composite laminates

Layers	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
Layer 1	🟥	🟥	🟥	🟨	🟨	🟨	🟨	🟥	🟨
Layer 2	🟨	🟨	🟥	🟥	🟨	🟨	🟨	🟨	🟨
Layer 3	🟨	🟨	🟨	🟨	🟨	🟨	🟥	🟨	🟥
Layer 4	🟨	🟥	🟨	🟨	🟥	🟨	🟨	🟨	🟨
Layer 5	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨
Layer 6	🟥	🟨	🟨	🟨	🟨	🟥	🟨	🟨	🟥
Layer 7	⬛	🟥	🟨	🟥	🟨	🟨	🟥	🟨	🟨
Layer 8	⬛	⬛	🟥	🟨	⬛	🟨	🟨	🟨	🟨
Layer 9	⬛	⬛	🟥	⬛	⬛	🟨	🟨	🟨	⬛
Layer 10	⬛	⬛	⬛	⬛	⬛	🟨	🟨	🟨	⬛
Layer 11	⬛	⬛	⬛	⬛	⬛	🟨	⬛	⬛	⬛

2.3 Classical Laminates Theory (CLT) and ABD Matrix

Roylance [12] proposed the classical laminate theory (CLT) which is a direct extension of the theory for bending of homogeneous plates with an allowance for in-plane tractions in additions to bending moments and for the varying stiffness of each ply in the analysis. There are a few assumptions made in this theory [13]; such as each lamina be in a state of plane stress and then to align the fiber direction with the fiber orientation angle relative to the laminate coordinate system, the lamina is rotated. The laminates is assumes to have perfectly bonded layer which means that there is no slip between adjacent layers. Each lamina is also considered to be a homogeneous layer, whether isotropic, orthotropic or transversely isotropic. CLT is applied in this research in order to analyze the elastic properties for the entire laminates, since the composite in the laminates behaved differently according to their mechanical properties. In order to define all these properties, laminate stiffness matrix (ABD matrix) consists of 6 x 6 matrix is served as a tool.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{pmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{1}$$

Components in matrix A carried extensional stiffness properties, matrix B consisted of extensional-bending coupling properties and matrix D comprised of bending stiffness properties of the lamina/laminates. The calculation for ABD matrix started with determination of reduced stiffness, Q by inserting the information of elastic constant for every layup followed by ABD matrix computation using the setup function in MATLAB. The matrix components for A_{ij}, B_{ij}, and D_{ij} are calculated using the following equations:

$$A_{ij} = \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(h_k - h_{k-1}\right); \qquad i,j = 1,2,6 \tag{2}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(h_k^2 - h_{k-1}^2\right); \qquad i,j = 1,2,6 \tag{3}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(h_k^3 - h_{k-1}^3\right); \qquad i,j = 1,2,6 \tag{4}$$

According to the layup arrangement in Figure 1, h_k and h_{k-1} referring to the position of the upper and lower surface of the k_{th} lamina from the center line. The uppermost layer became the first layer whilst the bottom layer counted as the last layer.

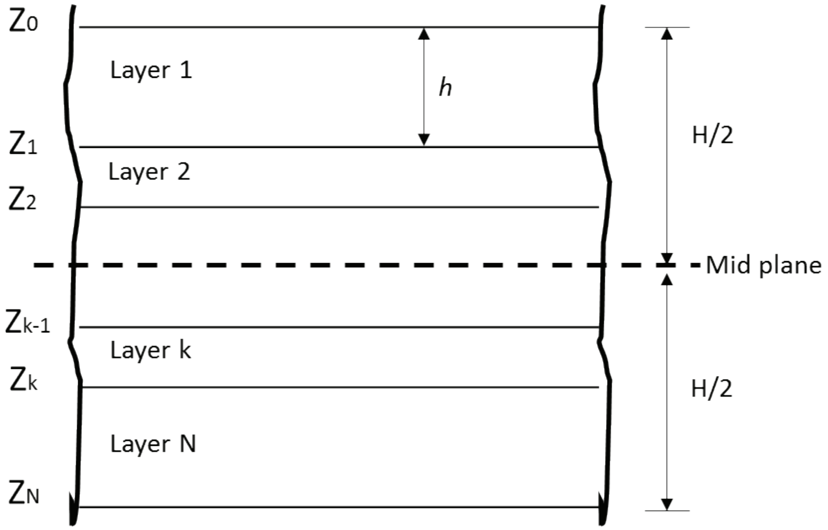


Figure 1: Layup arrangement in composite laminates

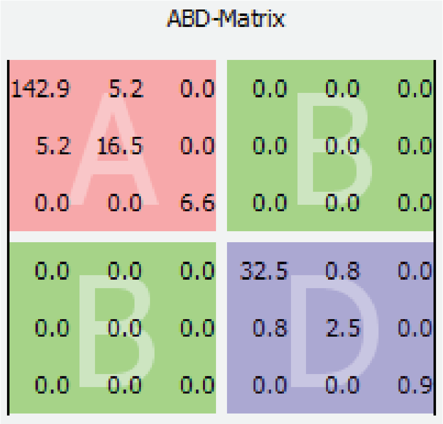
2.4 Analytical Modelling using MATLAB

Based on the layup configuration and elastic properties of CFRP and GFRP layup, the analytical modelling was developed. The method began by determining the positions of every layer from center line. The components of ABD matrix were calculated after the reduced stiffness, Q for every layup was determined. In order to determine the extensional stiffness, A matrix, and bending stiffness, D matrix for whole laminates, A and D matrix component was totaled up. B matrix for all cases was automatically zero due to the symmetrical configurations of laminates. Another composite laminates software, eLamX2, was used to validate the result from MATLAB calculations.

From the comparison between MATLAB and eLamX2, indicated that similar value of ABD matrix was achieved, thus suggested that the results is valid. Table 3 and Figure 2 show the comparison of result from MATLAB and eLamX2 for Case 1.

Table 3: ABD matrix calculation obtained from MATLAB

142.9	5.2	0.0	0.0	0.0	0.0
5.2	16.5	0.0	0.0	0.0	0.0
0.0	0.0	6.6	0.0	0.0	0.0
0.0	0.0	0.0	32.5	0.8	0.0
0.0	0.0	0.0	0.8	2.5	0.0
0.0	0.0	0.0	0.0	0.0	0.9



ABD-Matrix

142.9	5.2	0.0	0.0	0.0	0.0
5.2	16.5	0.0	0.0	0.0	0.0
0.0	0.0	6.6	0.0	0.0	0.0
0.0	0.0	0.0	32.5	0.8	0.0
0.0	0.0	0.0	0.8	2.5	0.0
0.0	0.0	0.0	0.0	0.0	0.9

Figure 2: ABD matrix calculation from eLamX2 software

2.5 Thickness Variant

The thickness for all cases is varied in the interest to design for parametric study and optimization. Three methods were applied in this study which is indicated in Table 4.

Table 4: Variant thickness for CFRP/GFRP

Thickness (mm)	1st Method (Baseline)	2nd Method	3rd Method
CFRP	0.45	0.225	0.45
GFRP	0.14	0.14	0.07

The original thickness is considered as the first method or the baseline for this study. The second method is the thickness of CFRP, which was reduced to half for all cases while maintaining the thickness of GFRP. Lastly, for the third method, the thickness of GFRP was reduced to half while maintaining the thickness of CFRP. The comparison in terms of ABD matrix is presented in table form and explained in discussion section.

3.0 RESULTS AND DISCUSSION

3.1 Effect of Stacking Sequence

Different configurations of unidirectional CFRP and GFRP in laminates composites were studied and analyzed in order to determine the arrangement that could offer the most optimum modulus of elasticity in longitudinal direction, E11. The ratio of CFRP and GFRP layup was varied ranging from single CFRP ply up to maximum four CFRP plies in different position in laminates. The arrangement of stacking sequence was considered to study the effect on extensional and bending stiffness. In terms of mechanical behavior, A11 is related with the deformation of composite laminates thus makes it equivalent to the modulus of elasticity in longitudinal direction, E11 [14]. E11 is calculated by

$$E_{11} = \frac{\text{Extensional Stiffness} - \text{Direction}, A_{11}}{\text{Total of Thickness of Laminates}, H} \tag{5}$$

The result in Figure 3 demonstrate the comparison of A11, D11 and E11 for all cases studied. A11 increased proportionally with the increased of CFRP layup in the laminates. Case 3 shows the highest value of E11 by 105.84 GPa whilst Case 6 have the least value of E11 by 66.72 GPa. This is due to the fact that the higher of extensional stiffness of CFRP contributes to the high extensional modulus of composite laminates than GFRP, thus having a more CFRP plies, give advantages in terms of elastic modulus for hybrid composite laminates but not in terms of manufacturing cost.

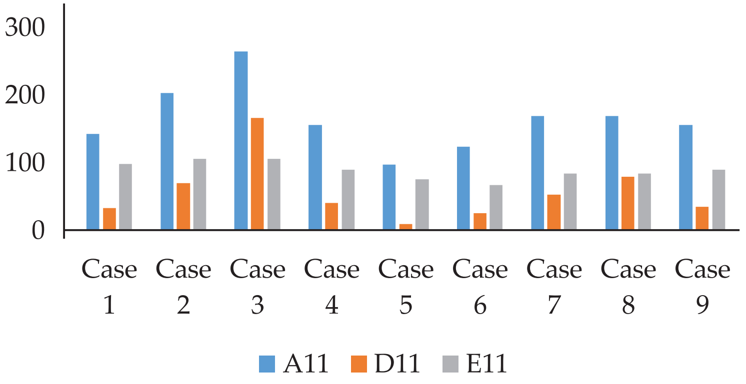


Figure 3: Comparison of A11, D11 and E11 for 9 cases

D11 on the other hand represent the bending stiffness in 1-direction, which is equivalent to the radius of the curvature of laminates [11]. Graphical result in Figure 3 shows that Case 4 and Case 9 as well as

Case 7 and Case 8 show same value of A11 but different in D11. The reasons is that all these cases (Case 4/Case 9 and Case 7/Case 8) had equal number of CFRP and GFRP plies to each other which contributed to equal extensional stiffness, A11 but different configuration of CFRP plies in the laminates. From D11 perspective, Case 4 is higher than Case 9 and Case 8 is higher than Case 7. This is because the position of CFRP plies at the outermost layer will increase the bending stiffness, at the same time result in smaller curvature in laminates. Placing CFRP layers at the compressive side was found to increase the bending strength of the hybrid composites when compared to placing the GFRP layers on the compressive side [15]. Since the mechanical properties of hybrid composite strongly depended on the reinforcing fiber position, thus, positioned CFRP plies at the outermost laminates is beneficial to composite laminates in terms of provides higher bending stiffness that resistance to bending deformation.

3.2 Effect of Variant Thickness

Modulus of elasticity, E11 is an important parameter in this study to analyze the performance of composite laminates for variant thickness. Table 5 summaries the thickness for all cases for all methods.

Table 5: Summary of thickness for all cases

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
First Method (Baseline) (mm)	1.46	1.92	2.5	1.74	1.29	1.85	2.02	2.02	1.74
Second Method (reduction of CFRP thickness) (mm)	1.01	1.235	1.6	1.29	1.0704	1.625	1.57	1.57	1.29
Third Method (reduction of GFRP thickness) (mm)	1.18	1.63	2.15	1.32	0.87	1.15	1.46	1.46	1.32

The result determined in terms of A11, D11 and E11 is tabulated as in Figure 4, Figure 5 and Figure 6. E11 in Figure 8 is lowering while CFRP thickness reduced to half. Since CFRP thickness is higher than GFRP, second method (CFRP reduced thickness) shows major decrease thus lead to lowering A11 and D11 of composite laminates.

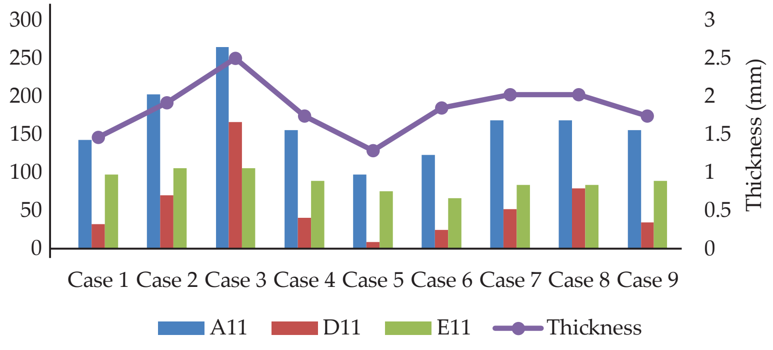


Figure 4: First method (baseline)

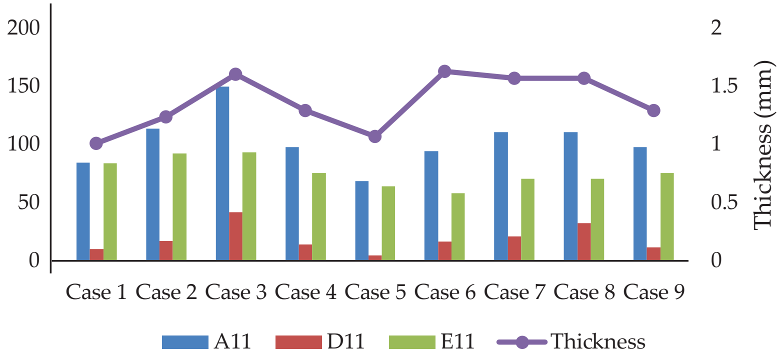


Figure 5: Second method (reduction of CFRP thickness)

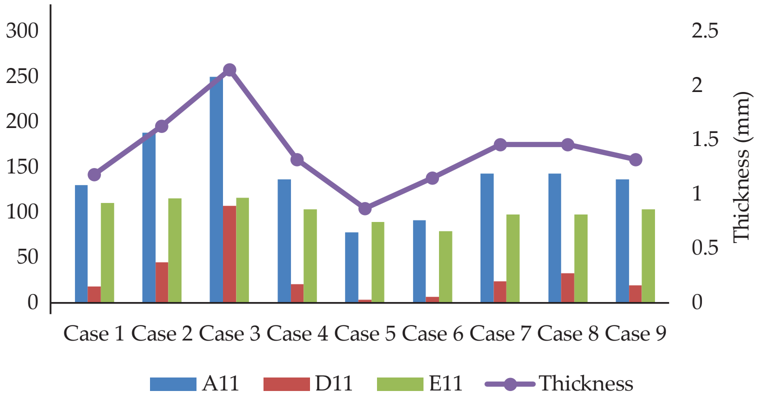


Figure 6: Third method (reduction of GFRP thickness)

Third method (Figure 9) has higher value of A11 compared to second method because thickness contribution from GFRP is smaller compared to CFRP. When an optimization in designing of composite laminates was conducted, GFRP reduced thickness is considered as an efficient method. However, for E11, third method had showed some improvements compared to baseline due to reducing the thickness of GFRP, the E11 for composite laminates will approach the elastic modulus of CFRP. CFRP reduced thickness for Case 2 is 12.79% and for Case 3 is 11.78% different from baseline. However, for GFRP reduced thickness, Case 2 is 8.66% and Case 3 is 8.99% higher from baseline. Generally, reduction of CFRP thickness in this study would reduce the extensional and bending stiffness of composite laminates, but reduction of GFRP thickness will slightly reduce the extensional and bending stiffness. This is because the CFRP was seen as the fiber dominated ply thickness thus introduces the largest detriment to the composite stiffness properties. Hence, the thinner GFRP layup is used, which will increase the elastic modulus of composite laminates.

4.0 CONCLUSION

The present study demonstrated variation of combined CFRP/GFRP composite and thickness where each case yields different set of mechanical properties. Some cases of hybrid composite laminates having similar number of CFRP, but having different arrangement will yield different D11 although possess equal value of A11. Increasing number of CFRP significantly improved the overall A11 of the combined composite and it distinguished that reducing single CFRP and increasing single GFRP (Case 2 and Case 3) will yield quite similar computation of E11 (within 3% deviation). Positioning of CFRP at outer/external layers and inner layers will not affect the computation value of A11 based on Case 7 and Case 8 as well as Case 4 and Case 9. From variation of thickness methodology, it is shown that CFRP/GFRP layup has an effect in terms of extensional stiffness, bending stiffness and modulus of elasticity of composite laminates. In this study, optimum thickness was determined for more successful mechanical properties application. Apart from lowering the production cost, the use of thinner GFRP layup may also increase the modulus of elasticity of composite laminates. From the present result, it is shown that by varying the stacking sequence of CFRP/GFRP and thickness in the composite laminates, the mechanical properties of the resulting hybrid material could be tailored according to the desire design requirement.

ACKNOWLEDGMENTS

Authors would like to thanks Universiti Teknikal Malaysia Melaka for MATLAB software and database for accessing literature. This research is funded under RAGS/1/2015/TK0/UTEM/03/17.

REFERENCES

[1] A.F. Ab Ghani and J. Mahmud, "Material Characterization of Hybrid Composite: Experimental Using Strain Gauge/DIC with Finite Element Modelling Macro/Micro Scale", *Key Engineering Materials*, vol. 740, pp. 31-40, 2017.

[2] X.H. Loh, M.A.M. Daud, and M.Z. Selamat, "Study on Fiber Length and Composition of Kenaf-Polypropylene (K-PP) Composite for Automobile Interior Parts", *Applied Mechanics and Materials*, vol. 695, pp. 36-39, 2015.

[3] R.T.D. Prabhakaran, H. Toftegaard, C.M. Markussen and B. Madsen, "Experimental and theoretical assessment of flexural properties of hybrid natural fibre composites", *Acta Mechanica*, vol. 225, no. 10, pp. 2775–2782, 2014.

[4] R.T.D. Prabhakaran, T.L. Andersen, C.M. Markussen, B. Madsen, and H. Lilholt, "Tensile and compression properties of hybrid composites-a comparative study", in 19th International Conference on Composite Materials, Montreal, Canada, 2013, pp. 1029-1035.

[5] Y. Swolfs, and I. Verpoest, and L. Gorbatikh, "Tensile failure of hybrid composites: measuring, predicting and understanding", *IOP Conference Series: Materials Science and Engineering*, vol. 139, no. 1, pp. 1-12, 2016.

[6] H. Mutsuyoshi, N.D. Hai, S. Asamoto, and T. Matsui, "Development of hybrid FRP composites as sustainable infrastructure materials", in US-Japan Workshop on Life Cycle Assessment of Sustainable Infrastructure Materials, Sapporo, Japan, 2009, pp. 1-9.

[7] Y. Nadir, and P. Nagarajan, M. Ameen and M.M. Arif, "Flexural stiffness and strength enhancement of horizontally glued laminated wood beams with GFRP and CFRP composite sheets", *Construction and Building Materials*, vol. 112, pp. 547-555, 2016.

[8] N. Attari, S. Amziane, and M. Chemrouk, "Flexural strengthening of concrete beams using CFRP, GFRP and hybrid FRP sheets", *Construction and Building Materials*, vol. 37, pp. 746-757, 2012.

[9] M. Nayeem Ahmed, P. Vijaya Kumar, H. K. Shivanand and S. B. Muzammil, "A Study on Effect of Variation of Thickness on Tensile Properties of Hybrid Polymer Composites (Glassfibre-Carbonfibre-Graphite) and GFRP Composites", *International Journal of Engineering Research and Applications*, vol. 3, no. 4, pp. 2015-2024, 2013.

[10] R. Kalhor, and S.W. Case, "The effect of FRP thickness on energy absorption of metal-FRP square tubes subjected to axial compressive loading", *Composite Structures*, vol. 130, pp. 44-50, 2015.

[11] M. Bruyneel, P. Duysinx, and C. Fleury, "Composite structures design for strength and stiffness with respect to ply thickness and/or fibers orientation", *Structural and Multidisciplinary Optimization*, vol. 31, no. 2, pp. 156-159.

[12] D. Roylance, *Laminated Composite Plates*. Cambridge: Massachusetts Institute of Technology, 2000.

[13] R.M. Jones, *Mechanics of composite materials, 2nd Edition*, New York: Taylor & Francis, 1975.

[14] N. Pan, J. Wu, K. Williams and Y. Wang, “Effects of floor coverings on posture steadiness and locomotion stability”, Technical Report, National Textile Center, 2006.

[15] I. A. Subagia, Y. Kim, L. D. Tijing, C. S. Kim and H. K. Shon, “Effect of stacking sequence on the flexural properties of hybrid composite reinforced with carbon and basalt fibers”, *Elsevier Composites: Part B*, vol. 58, pp. 251-258, 2014.