MINIMIZING PROCESS LEAD TIME FOR A SINGLE MACHINE USING THE DEVELOPED OPTIMAL BATCH SIZE EQUATIONS

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ABSTRACT: Manufacturing with an optimal batch size can significantly increase production performance. In the past, complicated techniques such as optimization models, simulation, queuing theory, and complex algorithms had been explored to solve for the optimal batch size. By applying those techniques, some customizations are needed when production factors such as demands and capacity change. It is even more difficult for a plant manager to customize the model when producing more than one type of products in a single machine. In the previous research studies, none of researchers proposed the equations that a plant manager can just put the numbers into and get the optimal solution. Therefore, the developed closed-form optimal batch size equations are proposed in this research. The formula can be easily used to assess the impact of changes in production volume. The purpose is to minimize process lead time. The developed optimal batch size equation can be applied to estimate the process lead time associated with the size of the batch when the demand is given. This research provides an illustration of proposed method with various parameters applied to different products. The optimal batch size is solved and the result verifies the effectiveness of the approach.

KEYWORDS: *Lead Time; Single Machine; Optimal; Batch Size*

1.0 INTRODUCTION

The manufacturing process with a large batch size usually has long lead time and low inventory turnover. Contrary to the Just-in-time concept, production with a large batch size carries large amount of finished goods inventory, and more safety stocks are required. In addition, more capital investments are tied to the cost of inventories.

During the past decades, numbers of researchers explored the approaches to calculate the optimal batch size which can improve the production efficiency.

In a study conducted by Parija and Sarker [1], it was found that the cost function and mixed integer programming were applied to find the optimal batch size with the objective to minimize the total costs of ordering and finished goods inventory. However, the limitation of their approach was that the cycle time interval between shipments were fixed. Wang and Chen [2] reported a new systematic procedure by modifying the cost function of Parija and Sarker to find the possible solutions. Then the optimal batch size was estimated by using a simulation model. Bertrand [3] published a paper in which finding the optimal batch size was explored by the approach of economic order quantity (EOQ) as well as queuing model. Wang and Sarker [4] investigated a new approach for optimal batch size in the Kanban system by applying mixed integer nonlinear programming. Ojha et al. [5] published an article in which they developed the model to optimize batch size with the focus on the manufacturing system with with quality assurance and rework issues. They developed the cost equation and evaluated the optimal ordering quantities. In another major study, Rau and OuYang [6] reported a mathematic model to solve for an optimal batch size when dealing with an integrated production–inventory policy in a supply chain. The main goal was to minimize the joint total cost incurred by the vendor and the buyer. Tielemans and Kuik [7] performed series of simulation experiments to compare the optimal batch size and the associated minimal lead time. However, the authors are only focused on a single item case. In the study conducted by Sohner and Schneeweiss [8], the approach of hierarchically integrated lot size optimization was introduced. The findings were computational feasible. The results were compared with those of the deterministic multi-level capacitated lot sizing model.

For the last few decades, very few studies have focused on batch sizing models that take lead time in to account in stochastic manufacturing system. In early stage, the manufacturing facility was modeled by a queueing system. The queueing problems in stochastic manufacturing system had been addressed in research introduced by Solberg [9], Shanthickumar and Buzacott [10], Whitt [11], and Karmarkar [12]. Shin et al. [13] proposed a stochastic model to solve for the optimal batch size with process and product variability. Millar and Yang [14] applied queueing network model to investigate the potential of batch sizing as a control variable for lead time performance. They also used discrete optimization via marginal analysis to solve the nonlinear batch sizing problem. Koo et al. [15] introduced a linear search algorithm to find the optimal throughput rate and the batch size at a bottle neck machine. Wang et al. [16] published their major development by proposing the chaotic-search-based self-organizing optimization approach to optimize the multistage batch scheduling problem. Yang et al. [17] applied iterative particle swarm optimization algorithm to solve for the batching optimization.

Much of the current literature on the optimal batch size issue pays particular attention to the techniques that need customizations when production factors such as demands and capacity change. It is also difficult for a plant manager to customize the model when producing more than one type of products in a single machine. To date, an easy-to-use formula to find the optimal batch size has still not been proposed. Therefore, this study presents a simple version of batch sizing model for a single process system that is easy to use. The objective is to derive a closed-form batch size equation that minimizes the overall process lead time.

2.0 ASSUMPTIONS AND NOTATIONS

In order to construct a mathematical model, the assumptions and notations are as follows;

2.1 Assumptions

There are 9 assumptions of the model as follows:

i. Demands are deterministic.
ii. Demands are constant over the evaluation period.
iii. If the demands are not constant during a long evaluation period, the evaluation period can be divided into many smaller periods. Each period has individual constant demands but not necessary be the same rate among different intervals. Therefore, the optimal batch size equation could be independently applied to each small period.
iv. Setup time is deterministic.
v. Setup time is an independent variable. In other words, set up time does not depend on what has been made before.
vi. There are no maximum limits on batch size.
vii. There are no minimum limits on batch size.

viii. There are no integrality restrictions.
ix. There are no non-negative restrictions.

2.2 Notations

Sets:

A = Set of all items.

Coefficients and parameters:

D_i = Total demand for item i.
D = Total demand of all items.
Note that $D = \sum_{i \in A} D_i$.
p_i = Processing time per unit of item i.
s_i = Setup time per batch of item i.
δ = Available setup time.
T = Available machine hours in the evaluation period

Decision variables:

B_i = Batch size of item i.
n_i = Total number of batches of item i required to produce D_i units.
Note that $n_i = \frac{D_i}{B_i}$ and $B_i = \frac{D_i}{n_i}$.

3.0 PROPOSED BATCH SIZE EQUATION THAT MINIMIZES OVERALL PROCESS LEAD TIME

In this model, the overall process lead time is estimated by the weighted average cycle time interval. If B_i is a feasible batch size of item i, the cycle time interval (CTI) of item i is explained in Equation (1).

$$CTI_i = \frac{D_i}{B_i} \tag{1}$$

Therefore, the weighted average cycle time interval is explained in Equation (2).

Weighted average $CTI = \frac{\sum_{i \in A} CTI_i D_i}{\sum_{i \in A} D_i}$ or can be simplified as

$$\text{Weighted average } CTI = \frac{1}{D}\sum_{i \in A} CTI_i D_i \tag{2}$$

The main goal is to find the feasible batch sizes of all products that minimize the overall process lead time. Regardless of the cost, the decision making is based on the production capacity, customer demands, set up time, and processing time.

The weighted average CTI in Equation (2) can be rewritten as the function of number of batches as shown in Equation (3).

$$\text{Weighted average } CTI_i = \frac{1}{D}\sum_{i \in A} CTI_i D_i \text{ or denoted by}$$

$$\text{Weighted average } CTI_i = \frac{1}{D}\sum_{i \in A} B_i = \frac{1}{D}\sum_{i \in A} \frac{D_i}{n_i} \tag{3}$$

When the demands are given, the total processing time is pre-deterministic. Therefore, the capacity constraint can be simplified. The total setup time in the actual production must not exceed the available setup time as shown in Equation (4).

$$\delta = T - \text{Total processing time or known as}$$

$$\delta = T - \sum_{i \in A} D_i p_i \tag{4}$$

However, the problem may be unfeasible if the available setup time is not sufficient to produce at least one batch of each item as shown in Equation (5) as

$$\delta < \sum_{i \in A} s_i \tag{5}$$

The number of batches would be fraction less than one or it could be a negative number. The objective function is defined in Equation (6). The constraint is defined in Equation (7).

$$\text{Objective function} = Min \frac{1}{D}\sum_{i \in A} CTI_i D_i \tag{6}$$

$$\text{Subject to } \sum_{i \in A} n_i s_i \le \delta \tag{7}$$

Note that there is only one constraint. Therefore, the Lagrangian multiplier technique can be applied to solve the problem [18]. The Lagrangian equation (L) is defined in Equation (8) such as

$$L = \frac{1}{D} \sum_{i \in A} \frac{D_i}{n_i} + \lambda \left(\sum_{i \in A} n_i s_i - \delta \right) \tag{8}$$

Next step is to minimize Equation (8) in order to solve for n_i.

First order conditions are given as

$$\frac{\partial L}{\partial \lambda} = -\frac{1}{D} \frac{D_i}{n_i^2} + \lambda s_i = 0 \text{; for all items} \tag{9}$$

$$\frac{\partial L}{\partial \lambda} = \sum_{i \in A} n_i s_i - \delta = 0 \tag{10}$$

Rearrange Equation (9) in order to move n_i^2 to the left hand side of the equation as shown in the following equation:

$$n_i^2 = \frac{D_i}{\lambda s_i D} \tag{11}$$

Then the total number of batches for item i (n_i) is defined in Equation (12).

$$n_i = \sqrt{\frac{D_i}{\lambda s_i D}} \tag{12}$$

Since n_i is a non-negative number, substitute n_i from Equation (12) in the Equation (10) in order to solve for λ as shown as

$$\lambda = \frac{1}{\delta^2 D} \left(\sum_{i \in A} \sqrt{D_i} s_i \right)^2 \tag{13}$$

From Equation (13), if at least one set up time per batch of item i (s_i) is positive, the optimal λ is always a positive number. As a result, the capacity constraint in the original problem is always the binding constraint as defined in Equation (14) such as

$$\sum_{i \in A} n_i s_i = \delta \tag{14}$$

Rearrange Equation (14), the total number of batches for item i (n_i) is defined as

$$n_i = \frac{\delta}{\sum_{i \in A} D_i s_i} \cdot \sqrt{\frac{D_i}{s_i}} \tag{15}$$

Therefore, the minimum batch size for item i (B_i) is defined in Equation (16).

$$B_i = \frac{D_i}{n_i} = \frac{\sum_{i \in A} \sqrt{D_i} s_i}{\delta} \cdot \sqrt{D_i s_i} \tag{16}$$

Although the numbers of batches obtained from the minimum batch size equation are usually not integer, these numbers are useful because they can be implied as the lower bound of the process lead time. In addition, they also imply the production frequency of the item in a production cycle. For example, if there were 20 batches of item A, 40 batches of item B, and 80 batches of item C to be produced, the production frequency ratio would be 1/20 : 1/40 : 1/80 which can be converted to 0.05 : 0.025 : 0.0125 or 4 : 2 : 1. It could be interpreted that item C would be produced every cycle. Item B would be produced every two cycles while item A would be produced every four cycles.

4.0 ILLUSTRATION

In order to demonstrate how to apply Equation (15) and Equation (16) into a real situation, a scenario is setup as follows. Assume that there are 5 products; A, B, C, D, and E, to be produced. The annual demands are 258, 1,105, 1,126, 1,130, and 500 units for product A, B, C, D, and E respectively. Processing time for each unit of A, B, C, D, and E are 0.25, 1.25, 1.8, 0.5, and 2 hours. The machine requires 20 hours for a setup before it can produce product A. Setup times for product B, C, D, and E are 30, 15, 25, 20 hours respectfully. It is assumed that the annual demand (D) of item A is 258 units. The setup time for producing item A is 20 hours and the processing time for each unit of item A is 0.25 hours. The production parameters are summarized in Table 1. From Table 1, total processing time required to produce all products is calculated as 0.25)(258) + (1.25)(1,105) + (1.8)(1,126) + (0.5)(1,130) +(2)(500) = 5,037.55 hours.

Table 1: Total demand and production parameters

Parameters	**Product A**	**Product B**	**Product C**	**Product D**	**Product E**
Processing time/unit p_i (hours)	0.25	1.25	1.80	0.50	2.00
Setup time / batch s_i (hours)	20.00	30.00	15.00	25.00	20.00
Annual demand D_i (units)	258.00	1,105.00	1,126.00	1,130.00	500.00

Table 2: Batch size with minimum lead time

Batches	**Product A**	**Product B**	**Product C**	**Product D**	**Product E**
Total batches (n_i) (batches in a year)	13.570	22.920	32.730	25.390	18.890
Batch size (B_i) (units for each batch)	19.020	48.200	34.410	44.500	26.480
Cycle time interval (days)	26.540	15.700	11.000	14.180	19.060

If there were 7,500 machine hours available in a year and the total processing time (excludes the set up time) were 5,037.55 hours, the available setup time would be equal to 7,500 – 5037.55 = 2,462.45 hours. Using Equations (15) and (16), the number of batches (n_i) and batch sizes (B_i) are shown in Table 2. Using Equation (1), the CTI for each product is also shown in Table 2. The minimized overall process lead time are then calculated using Equation (2). Therefore the overall process lead time to become 15.09 days in this case.

Assume that there are 360 working days in a year, the value of λ is calculated by using Equation (13). The λ value is equal to 0.000017. This implies that if total processing time decreased by 1 hour (which was equivalent to having an additional hour of available setup time), the process lead time would be decreased by 0.000017 year or 8.784 minutes.

5.0 APPLICATION OF BATCH SIZE EQUATIONS

By applying the optimal batch size equations explained above, some strategic issues such as the impact of changing total production volume could be determined. When manufacturing in a maximum capacity available and the demand is increased, it is inevitable to increase the batch sizes in order to obtain higher productivity. It is equivalent to reducing the number of batches. Therefore, the total setup time required is decreased. Producing in large batches may increase inventory level as well as the process lead time because each batch needs longer time to complete.

In contrast, when the demand is decreased, the total production volume is decreased. The total processing time required is also decreased. In other words, more setup time is available. As a result, manufacturing in smaller batches for each product is possible. This would lower inventory level as well as the process lead time.

For demonstrating an application of the batch size equations, the original example in Section 4.0 is used as a base case, where the percent change in demand is 0%. With different scenarios when demand decreases or increases incrementally by 5%, Figure 1 was created by using simple "What-if" analysis tool, "One-way Data Table" in Microsoft Excel. Figure 1 shows the significant changes in lead time and finished goods inventory level when the total demand volume changed. From Figure 1, the weighted average CTI or weighted by demand is calculated by using Equation (2) to measure the overall process cycle time interval.

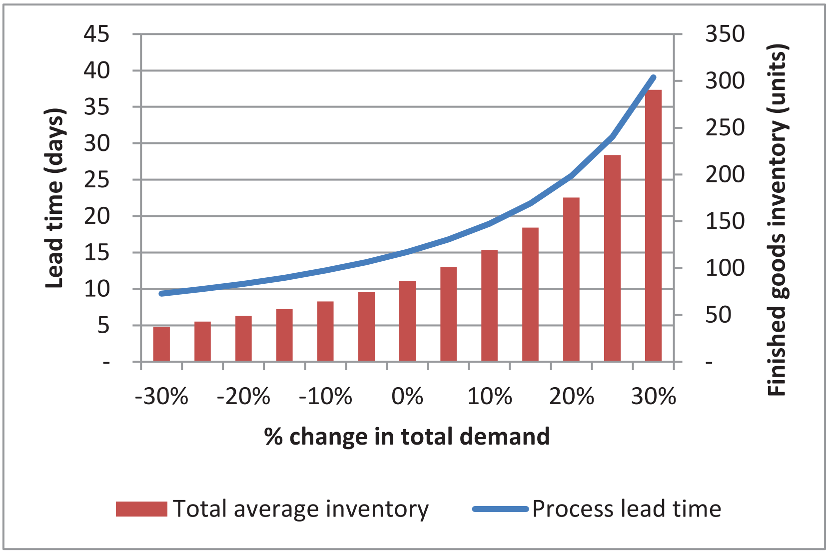

Figure 1: The impact of the change in total production volume to lead time and finished goods inventory level

Assume that the average finished goods inventory for each product is half of its batch size because the demand is constant. The percent change in total production is applied across all items. For instance, the demand for product i = (1+ %change) (original demand for product i).

The finding reported in Figure 1 shows that the weighted average CTI and total average finished goods inventory increase when the total production volume increases. The changes become significantly more sensitive when the production volume reaches the maximum capacity. Interestingly, if the information on additional sales revenue is available, this trade-off curve can be applied to perform the cost-benefit analysis to determine whether the additional sales revenues from adding more production volumes worth the loss in flexibility or the increase in lead time and more assets will be tied to the increase in finished goods inventory.

6.0 CONCLUSION

This research proposes the optimal batch size equation which can be applied to minimize process lead time. The optimal batch size equation is the generalized version of the typical process and it can be applied to estimate the process lead time associated with the size of the batch when the demand is given.

The most striking result to emerge from this approach is the value of λ, which is the dual variable associated with the capacity constraint in the original problem. It can be interpreted as a "shadow price" of available setup time that can be used to quickly estimate the impact of overall processing time reduction that is equivalent to obtaining additional available setup time to overall process lead time. The optimal batch size equation is also applied to explain the impact of increasing production volume. This approach is user friendlier compared to those techniques most researchers had proposed in the past. The result shown in the illustration section verifies the effectiveness of the approach.

However, this research has thrown up many questions in need of further investigation. Further works need to be done to establish the optimal batch size equation with a different objectives. Future research could be involved with inventory cost. Deriving and finding the optimal batch size equation which minimizes the setup cost and holding cost should be explored.

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