A DATA-DRIVEN PID CONTROLLER FOR FLEXIBLE JOINT MANIPULATOR USING NORMALIZED SIMULTANEOUS PERTURBATION STOCHASTIC APPROXIMATION

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ABSTRACT: This paper presents a data-driven PID controller based on Normalized Simultaneous Perturbation Stochastic Approximation (SPSA). Initially, an unstable convergence of conventional SPSA is illustrated, which motivate us to introduce its improved version. The unstable convergence always happened in the data-driven controller tuning, when the closed-loop control system became unstable. In the case of flexible joint manipulator, it will exhibit unstable tip angular position with high magnitude of vibration. Here, the conventional SPSA is modified by introducing a normalized gradient approximation to update the design variable. To be more specific, each measurement of the cost function from the perturbations is normalized to the maximum cost function measurement at the current iteration. As a result, this improvement is expected to avoid the updated control parameter from producing an unstable control performance. The effectiveness of the normalized SPSA is tested to the data-driven PID control scheme of a flexible joint plant. The simulation result shows that the data-driven controller tuning using the normalized SPSA is able to provide a stable convergence with 76.68 % improvement in average cost function. Moreover, it also exhibits lower average and best values for both norms of error and input performances as compared to the existing modified SPSA.

KEYWORDS: *Data-Driven; Improved Stochastic; PID Controller Tuning; SPSA*

1.0 INTRODUCTION

The selection of the optimization tools is important to obtain a better control performance in the data-driven controller tuning. In general, it consists of two groups of optimization approaches: population-based optimization and trajectory-based optimization. Recently, population-based optimization approaches were widely reported in data-driven control frameworks, such as in tuning the controller of under-actuated systems, manufacturing and industrial plants, transportation systems, and alternative energy plants. Nevertheless, these population-based tools need large computation time to reach convergence state. This is because the number of measured cost functions at each iteration is proportional to the number of populations. Meanwhile, recent applications of the trajectory-based optimization in the data-driven control scheme mainly include the parameters tuning in the PID controller [1], fuzzy logic controller [2] and neural network controller [3]. These methods focus on exploiting and enhancing a single candidate solution by perturbing its design parameter elements with random values. Hence, these approaches are expected to produce less computation time than the population-based methods.

The simultaneous perturbation stochastic approximation [4], which is in the class of trajectory-based optimization, is a highly effective algorithm that approximates the gradient based on only two cost function measurements. By focusing on the practical application, the standard SPSA algorithm is applicable to various engineering problems, such as feature selection [5], traffic systems [6], system identifications [7], neural networks [8], and multi-agent systems [9]. Although it is useful for a variety of applications, the original SPSA has also been extended to several versions to solve different problems. In [10], it has been shown that a one measurement SPSA is better than the conventional two measurements SPSA for optimizing the design parameter of non-stationary systems. In order to accelerate the convergence of the conventional SPSA, a second-order version of SPSA or an adaptive SPSA has been proposed in [11]. The idea involves two parallel recursions, which are the estimation of a design parameter and the estimation of the Hessian matrix of the design parameter. Meanwhile, the global convergence SPSA has been presented by [12] to avoid a local minimum problem, especially in the early iterations of the conventional SPSA. This is achieved by introducing an injected noise in the updated law. In [13], the gradient is approximated using perturbed and unperturbed measurements, which is called one-sided SPSA. This idea can reduce the number of measurements as well as preserve the normal convergence of the original SPSA. A multi-resolution SPSA has been developed to increase the convergence speed of the conventional SPSA, especially for high-dimensional tuning problems. The idea of a multi-resolution SPSA means that the tuning operation is separated into some phases with a difference in the design parameter's dimension.

In the context of a data-driven control framework, the SPSA algorithm can become a useful optimization tool. However, the conventional SPSA cannot be straightforwardly implemented for tuning the control parameter. This is because there is a probability that the updated control parameter in the conventional SPSA grows rapidly and yields an unstable solution. Moreover, it is difficult to guarantee that the tuning control parameter produces a stable closed-loop system in all conditions since the closed-form expression of the control cost function is not known. Recently, the work in [14-15] have proposed a norm-limited SPSA by introducing a saturation function to limit the updated control parameter. Despite of producing a stable convergence, the solution was only applicable to some data-driven control problems. Furthermore, the saturation function will bound the exploration of finding the optimal control parameter. Therefore, it would be beneficial to further enhance the conventional SPSA in producing a stable convergence with a better optimal solution.

This paper proposes an improved SPSA based on normalized gradient approximation. Firstly, an unstable convergence of the conventional SPSA is illustrated. Then, the proposed normalized SPSA is presented. Finally, the proposed normalized SPSA is tested to the data-driven PID controller in flexible joint control problem.

Notation: The set of real numbers and positive real numbers are denoted by $\mathbf{R}$, $\mathbf{R}_+$, respectively. We define $\mathbf{0}$ and $\mathbf{1}$ as the vector whose all elements are zero and one, respectively.

2.0 METHODOLOGY

Firstly, the conventional SPSA algorithm is explained in this section. Then, the unstable convergence of the SPSA algorithm is shown. Note that we adopt the same example as in [15] to verify our proposed method.

2.1 Conventional SPSA

Let $f : \mathbf{R}^n \to \mathbf{R}$ be the cost function and $z \in \mathbf{R}^n$ is the design variable. Then, a standard optimization problem is written by

$$\min_{z \in \mathbf{R}^n} f(z) \tag{1}$$

The SPSA algorithm updates z iteratively by using an updated equation as follow

$$z_{k+1} = z_k - a_k g(z_k) \tag{2}$$

where $z_k \in \mathbf{R}^n$ is the design variable at k iteration, $a_k \in \mathbf{R}_+$ is the gain sequence and $g(z_k) \in \mathbf{R}^n$ is the gradient approximation given by

$$g(z_k) = \begin{bmatrix} \dfrac{f(z_k + c_k\Delta_k) - f(z_k - c_k\Delta_k)}{2c_k\Delta_{1k}} \\ \dfrac{f(z_k + c_k\Delta_k) - f(z_k - c_k\Delta_k)}{2c_k\Delta_{2k}} \\ \vdots \\ \dfrac{f(z_k + c_k\Delta_k) - f(z_k - c_k\Delta_k)}{2c_k\Delta_{nk}} \end{bmatrix} \tag{3}$$

In Equation (3), $c_k \in \mathbf{R}_+$ is another gain sequence, $\Delta_k \in \mathbf{R}^n$ is a random perturbation vector and $\Delta_{ik} \in \mathbf{R}$ is the i-th element of vector, $\Delta_k \in \mathbf{R}^n$.

The main concept of the SPSA algorithm is that the expectation of $g(z_k) \in \mathbf{R}^n$ closely identical to the gradient of the cost function f, namely, $\frac{\partial f}{\partial z} z_k$ and thus Equation (3) corresponds to a kind of stochastic steepest descent.

2.2 Unstable Convergence of Conventional SPSA

Based on the conventional SPSA algorithm, we illustrate its unstable convergence by using the same example from [15].

Consider the cost function such as

$$f(z) = ((z - \mathbf{1})^{\mathrm{T}}(z - \mathbf{1}))^3 \tag{4}$$

that has a global minimum point at $\mathbf{z} = \mathbf{1}$ for n = 10. Here, we set $a_k = 0.05/(k+200)^{0.602}$, $c_k = 0.01/(k+1)^{0.101}$ and Δ_k is generated from random Bernoulli vector for $z_0 = \mathbf{0}$. Figure 1 depicts the cost function convergence response of the conventional SPSA algorithm after 30 iterations. It indicates that the conventional SPSA algorithm diverge to the maximum value. This finding implies that the conventional SPSA algorithm does not guarantees to give a stable convergence. Hence, it encourages us to introduce a modified SPSA algorithm.

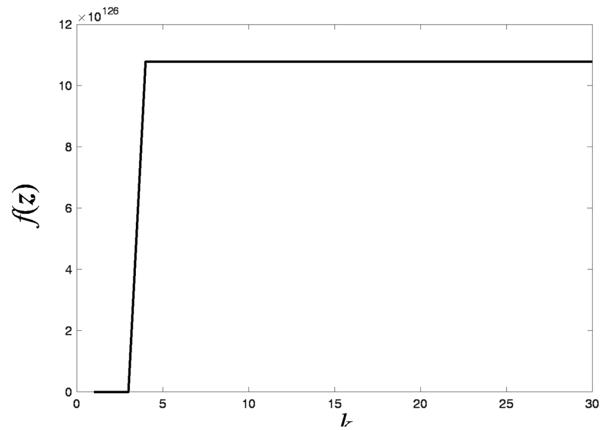

Figure 1: The f(z) convergence response of the conventional SPSA

2.3 Normalized SPSA

This section demonstrates the solution of the unstable convergence of the conventional SPSA algorithm. For simplicity of equation representation, let $f(z_k + c_k\Delta_k)$ and $f(z_k - c_k\Delta_k)$ are denoted by $f(\mathbf{z}^+)$ and $f(\mathbf{z}^-)$, respectively. Here we improve the updated design variable in Equation (2) becomes

$$\mathrm{z}_{k+1} = \mathrm{z}_k - a_k \tilde{g}(\mathrm{z}_k) \tag{5}$$

where

$$\tilde{g}(z_k) = \begin{bmatrix} \dfrac{h(\tilde{f}(z^+), \tilde{f}(z^-))}{2c_k \Delta_{1k}} \\ \dfrac{h(\tilde{f}(z^+), \tilde{f}(z^-))}{2c_k \Delta_{2k}} \\ \vdots \\ \dfrac{h(\tilde{f}(z^+), \tilde{f}(z^-))}{2c_k \Delta_{nk}} \end{bmatrix} \quad (6)$$

In Equation (6), $h(\tilde{f}(z^+), \tilde{f}(z^-))$ is a function given by

$$h(\tilde{f}(z^+), \tilde{f}(z^-)) = \begin{cases} 1 & \text{if } \tilde{f}(z^+) = \tilde{f}(z^-), \\ \tilde{f}(z^+) - \tilde{f}(z^-) & \text{if } \tilde{f}(z^+) \neq \tilde{f}(z^-), \end{cases} \quad (7)$$

where $\tilde{f}(z^{\pm})$ are normalized cost functions which can be written as

$$\tilde{f}(z^{\pm}) = \frac{f(z^{\pm})}{\max\{f(z^+), f(z^-)\}} \quad (8)$$

In order to illustrate the successfulness of the normalized SPSA algorithm, we choose again the cost function in Equation (4). Here, we apply the proposed algorithm in Equation (5) using the same condition as conventional SPSA, except for $a_k = 1.2/(k+200)^{0.602}$. Figure 2 shows the cost function convergence response of the normalized SPSA. It proves that the proposed method successfully minimizes the given cost function and solves the unstable convergence problem.

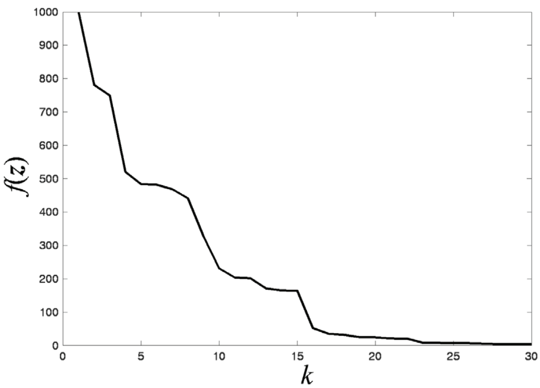

Figure 2: The f(z) convergence response of the normalized SPSA

3.0 RESULTS AND DISCUSSION

In this section, we evaluate the effectiveness of our normalized SPSA to the data-driven PID control design of flexible joint manipulator. In this control problem, the researcher is interested to solve the vibration issue in a lightweight robotic arm, such as flexible joint manipulator [16]. Consider the PID control system of the flexible joint plant in Figure 3, where $r(t)$ is the tip angular reference, $\theta(t)$ is the actual tip angular position, $\alpha(t)$ is the deflection angle of flexible link and $u(t)$ is the control input. The flexible joint plant G, which is adopted from [16], is given by

$$G := \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{9}$$

where A, B, C and D are taken from [16]. The details of the flexible joint plant can also be referred to [16]. The PID controller is given by

$$K_i(s) = P_i\left(1 + \frac{1}{I_i s} + \frac{D_i s}{1 + (D_i / N_i)s}\right) \tag{10}$$

for i =1, 2, where $P_i \in \mathbf{R}$, $I_i \in \mathbf{R}$, $D_i \in \mathbf{R}$ and $N_i \in \mathbf{R}$ are the proportional gain, integral time, derivative time and filter coefficient, respectively.

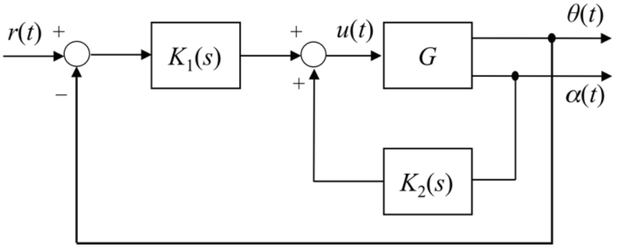

Figure 3: PID control system block diagram for flexible joint plant

In this flexible joint control problem, it is required that the tip angular position is rotated to a given reference position or tracking while producing very minimum oscillation of deflection angle and reasonable control input. Therefore, the performance index is yielded as

$$J(\psi) = 400\int_0^4 \left| r(t) - \theta(t) \right|^2 dt + 400\int_0^4 \left| \alpha(t) \right|^2 dt + \int_0^4 \left| u(t) \right|^2 dt \tag{11}$$

where $\psi = [P_1\ I_1\ D_1\ N_1\ P_2\ I_2\ D_2\ N_2]^{\mathrm{T}} \in \mathbf{R}^8$ for

$$r(t) = \begin{cases} 50\,t, & 0 \le t \le 1, \\ 50, & 1 \le t \le 4. \end{cases} \tag{12}$$

Remark: Note that the reference tip angular position in Equation (12) is based on ramp function from 0 to 1 sec before it goes to 50 degrees after 1 sec. This kind of input is applied to observe the successfulness of the data-driven PID based on normalized SPSA in tracking the given reference tip angular especially during the ramp reference situation. Moreover, this type of input is normally used in real application to reduce the magnitude of vibration if the normal unit step is applied.

Next, we perform the normalized SPSA algorithm in Equation (5) by setting f as J and $z_i = \log\psi_i\ (i = 1, 2, \ldots, 8)$, where z_i is i-th element of $\boldsymbol{z}$. Note that the logarithmic scale is applied to the design variable to speed up the exploration of finding the optimal solution. We set the gain sequences $a_k = 1.5/(k + 24)^{0.7}$, $c_k = 0.2/(k + 1)^{0.1}$, for $\boldsymbol{z}_0$ = [0.5 1.0 0.0 1.0 0.0 1.0 0.0 2.0]. In this study, 30 trials are considered to evaluate the statistical performances of normalized SPSA. In particular, the statistical performances with regard to average, best, worst and standard deviation (Std.) of the cost function, norm of error and norm of input are considered.

Figure 4 shows the best cost function convergence response (from 30 trials) of the normalized SPSA for 200 iterations. It demonstrates that the normalized SPSA algorithm is able to provide stable convergence in data-driven PID control tuning of flexible joint plant. The optimal design variable is $\boldsymbol{Z}_{200}$ = [1.33 1.21 -0.77 2.32 -0.73 1.61 0.37 0.97], which is corresponded to the optimal PID parameters ψ_{200} = [21.50 16.23 0.17 210.20 0.19 40.41 2.37 9.27]. The responses of the $\theta(t)$, $\alpha(t)$ and $u(t)$ are illustrated in Figures 5, 6 and 7, respectively. Here, the red-dotted line corresponds to the response of the controller based on initial PID parameters (k = 0), while the straight black line refers to optimal PID parameters ($k = 200$). In Figure 5, it shows that the data-driven PID based normalized SPSA successfully improves the tip angular position tracking, with very minimal overshoot and almost zero steady state error. In terms of deflection angle (Figure 6), the optimal PID controller is able to minimize the oscillation of deflection angle faster than the initial PID controller, which is within 2 second. However, it produces quite higher magnitude of deflection, which is from -1.9 degree to 2.4 degree, as compared to initial PID controller. Similarly, the control input of optimal PID controllers produces lower settling time with higher magnitude of input as compared to initial PID parameter.

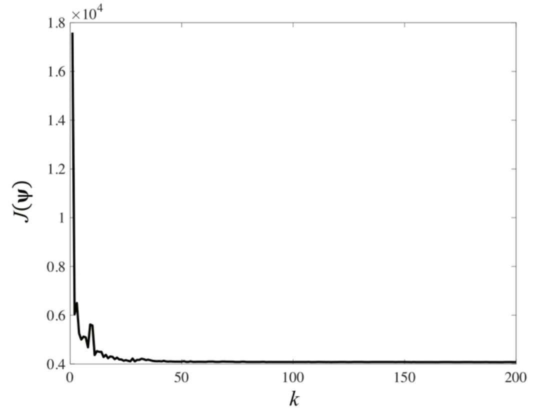

Figure 4: The $J(\psi)$ convergence response of the normalized SPSA

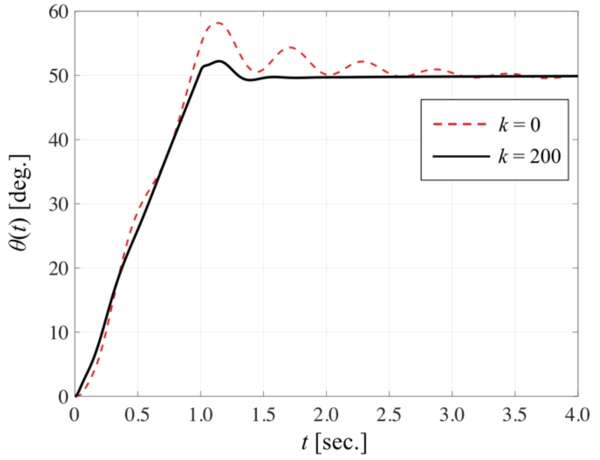

Figure 5: The response of the $\theta(t)$ using the normalized SPSA

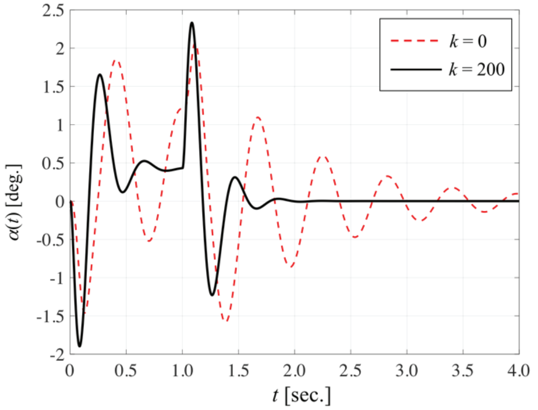

Figure 6: The response of the $\alpha(t)$ using the normalized SPSA

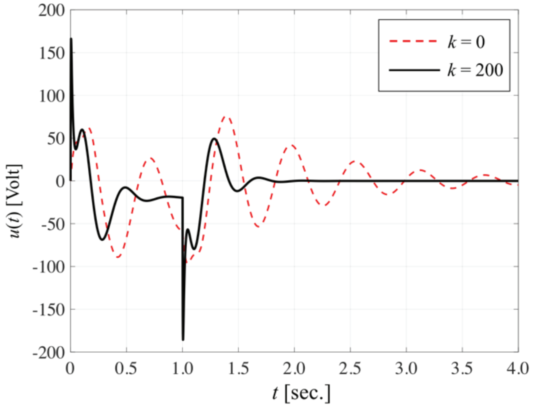

Figure 7: The response of the $u(t)$ using the normalized SPSA

Table 1: Statistical performances between normalized SPSA and norm-limited SPSA [14]

Algorithm		Norm-limited SPSA [14]	Normalized SPSA
$J(\psi)$ (x10^3)	Average	4.1203	**4.1027**
	Best	4.1013	**4.0717**
	Worst	4.1493	**4.1486**
	Std.	**0.0126**	0.0191
$\int_0^4 \lvert r(t)-\theta(t)\rvert^2 dt + \int_0^4 \lvert\alpha(t)\rvert^2 dt$	Average	3.3717	**3.3521**
	Best	3.3039	**3.2457**
	Worst	**3.4108**	3.4306
	Std.	**0.0257**	0.0303
$\int_0^4 \lvert u(t)\rvert^2 dt$ (x 10^3)	Average	2.7716	**2.7618**
	Best	2.7486	**2.7363**
	Worst	**2.7987**	2.8180
	Std.	**0.0126**	0.0187

In addition, we also compare the performance of our data-driven PID based on normalized SPSA with the data-driven PID based on norm-limited SPSA in [14]. Table 1 shows the statistical analysis of the performance index for both methods after 30 trials. Note that the bold values in Table 1 indicate the best performance. It shows that the normalized SPSA produce slightly better average and best values in the performance index, tracking error and control input compared to the norm-limited SPSA. Hence, it proves that the proposed SPSA is able to provide better control accuracy than the norm-limited SPSA.

4.0 CONCLUSION

In this paper, an improved SPSA based on normalized gradient approximation has been proposed to solve unstable convergence problem in the conventional SPSA. The effectiveness of the normalized SPSA has been validated to data-driven PID control of a flexible joint manipulator plant. The results show that the proposed SPSA is able to produce a stable convergence in tuning the given PID controller. Moreover, it also provides better control accuracy than the norm-limited SPSA. In the future, the normalized SPSA will be considered for tuning a controller of complex MIMO system.

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