AUGMENTATION OF SIMPLEX ALGORITHM FOR LINEAR PROGRAMMING PROBLEM TO ENHANCE COMPUTATIONAL PERFORMANCE

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ABSTRACT: Linear programming (LP) has been seen as a tool to solve problem in mathematical way with various methods to perform the solution. Simplex method is one of pioneer methods in dealing with linear problem in LP. It involves step-by-step works towards the solution in its algorithm. Due to this distinctiveness, it has brought up interest in others and few studies were done by researchers to come out with augmentation study in enhancing computational performance of Simplex method in terms of initialization, iteration and termination. In this paper, three studies were recognized in augmenting Simplex algorithm namely Basic Line Search Algorithm (BLSA), ε-Optimality Search Direction (ε-OSD) and Quick Simplex Method (QSM). Next,theoretical backgrounds were developed as a foundation to generate prototypes of new methodologies from the combination of these three methods. Then, the prototypes underwent a mathematical computation before the verification and validation procedures for reliability and efficiency of the new methodologies. The generation of the new methodologies eventually overcoming the pitfalls of the computational performance and striving toward its completion.

KEYWORDS: *Linear programming, Simplex method, augmentation of Simplex method*

1.0 INTRODUCTION

Linear programming (LP) is a mathematical model with aptitude to solve various sizes of problem [1]. The discovery of this applied mathematics has made it the most technique used over a large area [2]. A wide-ranging of study and exploration of LP is based on theories, algorithms and applications [3]. The utilization of LP is not constrained, thus, improvement of ideas has been broadly utilized [5]. Thus, LP has given mankind the ability to state general goals and laid out a path of detailed decisions to achieve the best goals [6]. In LP problem studies, there are various methods to perform the solution and one of the earliest is Simplex method. It is the most constructive tool to educate, work out LP problems [14], solve and serve as a foundation to design other methods [21]. All these specialities have triggered the interests to study the augmentation of Simplex method. In this paper, three methods have been recognized as works of Simplex method's augmentation to improve the computational performance pitfalls in terms of initialization, iteration and termination of Simplex algorithm. The three methods are BLSA, ε-OSD and QSM which are further discussed in literature review. Next, new methodologies from the two combination of pairs of these three studies are proposed to enhance the computational performance.

2.0 LITERATURE REVIEW

2.1 Mathematical Modeling with Linear Programming

LP that frequently works with types of problem that are expressed in a linear form [7] and its mathematical programming has consolidated philosophy (hypothesis), calculation (experimentation) and sober mindedness whereby its main instrument has dependably been LP [8]. It is perceived by numerous operations to tackle reasonable models from an expansive range of issue territories [9]. Specifically, problem to be optimized is having its objectives formulated in linear function [10,16] and liable to linear equality and linear inequality of constraints [11]. In performing Simplex algorithm, LP's mathematical formulation is performed first to present the problem's attributes namely decision variables, objective function and constraints as shown in Table 1.

Table 1: Basic components of LP model [13]

Attribute	Definition	Example of formulation
Decision variables	Matters that seek to determine.	Variables are statements of matters that seek to be determined.
Objective function	Matters that need to optimize. A goal either to be minimized or maximized.	$\boldsymbol{Maximize\ Z = \sum_{j=1}^{n} c_j\, x_j}$ where $x_1, x_2, \ldots, x_n$ are the problem variables, $c_1, c_2, \ldots, c_n$ are constants so-called the cost coefficient.
Constraints	Matters that the solution must satisfy.	$\sum_{j=1}^{n} a_{ij}x_j \le b_i, (i = 1, 2, \ldots, m)$ $x_j \in R(j = 1, 2, \ldots, n)$ $x_1, x_2 \ge 0$ where $b_1, b_2, \ldots, b_m$ are the resource value coefficients, and a_{ij}, (i = 1, 2,…, m) (j = 1, 2,…, n) are the constraint coefficients. Restriction $x_1, x_2 \ge 0$ are referred to as nonnegativity constraints.

2.2 The Conventional Simplex Method

Simplex technique has proven to be the most effective in solving LP problems [11]. It works with algorithm which moves from vertex to vertex of the primal feasible region until it reaches an optimal solution and each vertex corresponds to a basic feasible solution [12]. There is a polyhedron with flat faces having $n + 1$ vertices in n-dimensional simplex where n is number of independent variables [15]. In application, when there is scarcity in problem of resources' utilization, for instance, labor, materials, machines, tools or capital, then the needs arise for this optimization technique [20]. A brief review of the conventional simplex algorithm is as presented in Figure 1.

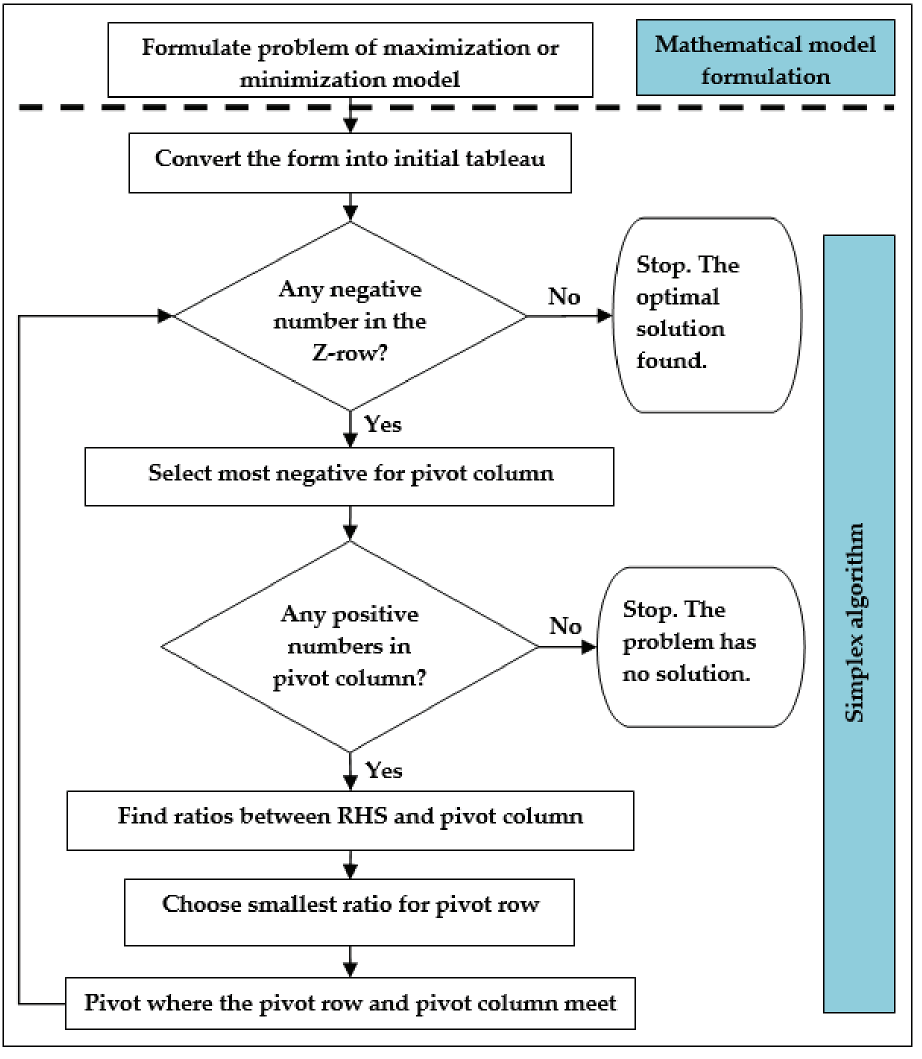

Figure 1: Process flow of conventional Simplex algorithm [22]

2.3 Computation Performance Pitfalls in Simplex Algorithm

A study found that there are three pitfalls of computational performance when performing the Simplex algorithm [17] involving inititalization, iteration and termination, as illustrated in Figure 2. The pitfall in initialization is a problem of finding initial feasible solution to start the Simplex method. It is compulsory to decide the initial point and it is known that the use of Simplex algorithm requires at least one basic feasible solution [23]. Second pitfall is iteration, which is difficulties in choosing, entering or leaving variable. Iteration study in Simplex method is an active area of LP to improve its pivoting selection strategies efficiently and to construct initial Simplex tableau [24]. The third pitfall is termination, whereby to ensure the algorithm terminates and does not merely continue through endless sequence of iterations without ever reaching an optimal solution. This is due to the conventional Simplex method which notably increases the number of variables and iterations [25], thus, the termination is getting tedious. Hence, this scenario triggers the interests in augmentation study of Simplex algorithm.

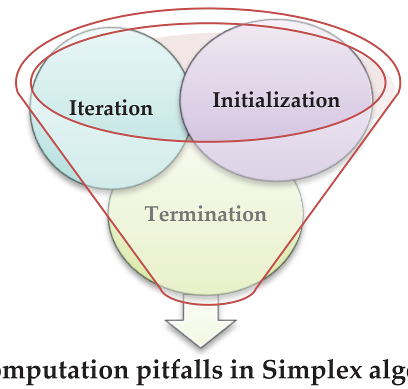

Figure 2: Three major pitfalls solving LP problems by Simplex method [17]

2.4 Augmentation Study of Simplex Algorithm

The augmentation works of Simplex method arise rapidly with a lot of variants type of LP problems. The focused area here is on the augmentation study with linear problems. These three studies were selected based on their problems of study which was linear problem and the augmented works performed had taken into account the computational performance. Detail review of these three methods are discussed below.

2.4.1 Basic Line Search Algorithm (BLSA)

The idea of BLSA method [4] is that instead of moving from 0-dimensional face (vertex) to an improved one, it moves from 1-dimensional face (edge) to an improved one. It skips some adjacent vertices and converges faster than the simplex method. As illustrated in Figure 3, the LP problem is in the 2-dimensional space, where the hatched area is the feasible set, and x^* is the optimal solution. Suppose the Simplex method starts from the origin O, then the algorithm has to visit the sequences of vertices O→A→B→C→D or O→F→E→D to reach the optimal solution x^*. However, if BLSA method is used, starting from the origin O, along line OA or OF, it moves to line CD or DE and reaches the optimal solution by only one iteration as described in Figure 4. This is the reason BLSA has improvised the computational performance of termination pitfalls as this method confirms the location of optimal solution.

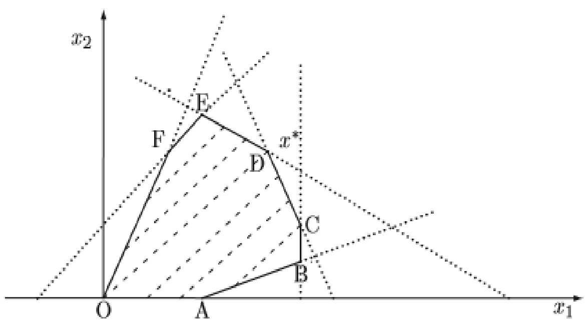

Figure 3: Illustration example of polyhedron X [4]

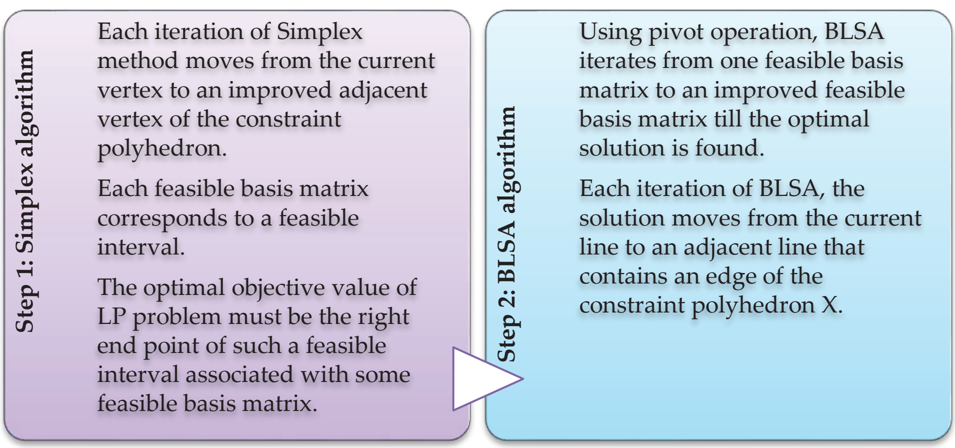

Figure 4: Illustration of the idea of BLSA algorithm [4]

2.4.2 ε-Optimality Search Direction (ε-OSD)

The ε-OSD method [18] was from the idea on solving LP medium-problem size by combining Simplex and interior point method. Apprently, the interior point methods dominate Simplex-based methods only in a solution of very large scale LPs. Hence, ε-OSD intends to start initial basic feasible solution near the optimal point, as illustrated in Figure 5. A loop of 7 steps is set for determining a basic feasible solution to start the Simplex method. The loop implements gradient projection method is used to obtain a corner point of feasible solution. Before obtaining a basic feasible solution, it is easy to check the maximum number of iterations in the loop.

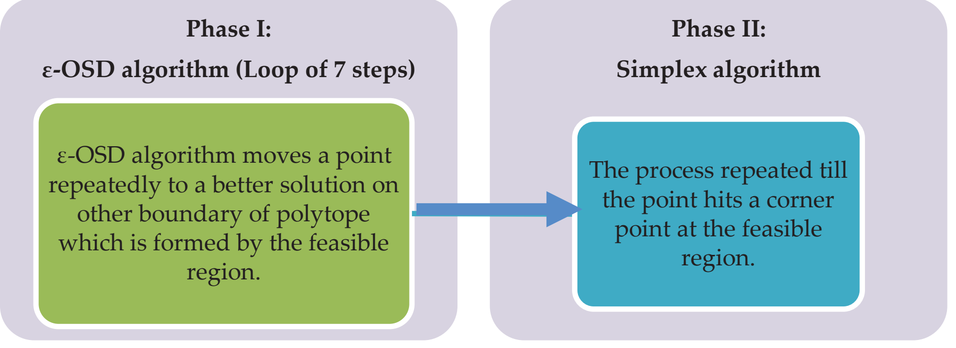

Figure 5: Illustration of the idea of ε-OSD algorithm [18]

The ε-OSD algorithm helps to posses a better worst-case complexity bound. Its computational performance has been explored on some random test problems and the result indicates that it helps Simplex method in reducing the number of iterations about 40%, as shown in Table 2. Thus, this method has improvised the computational performance of initialization and iteration pitfalls.

Table 2: Iteration number comparison between Simplex and Simplex with ε-OSD method [18]

Number of constraints	Number of variable	Average total iteration number	
		Simplex method	Simplex + ε-OSD method
10	10	6	3
20	20	17	11
30	30	31	19

2.4.3 Quick Simplex Method (QSM)

A method is designed [19] based on the decision of choosing, entering and leaving variable in conventional Simplex method. It is known that, the current Simplex method is rather inconvenient in handling degeneracy and cycling type of problems because the choice of vectors, entering and leaving, plays an important role. The degeneracy occurs when there is a tie for outgoing vector. The possibility of cycling is crucial only if the current basic feasible solution has more than one variable zero. Thus, in current practice of Simplex method, when a tie for entering the vector arises, the vector with the lowest index is selected.

However, QSM method attempts to replace more than one basic variable simultaneously. The power of QSM method lies in getting rid of the tie, especially in the degeneracy type of problems. This method may involve less iteration than in the Simplex method or at the most an equal number to it. Few problems of computation are performed to compare the initialization and iteration performance as shown in Table 3. Therefore, this method has improvised the computational performance of initialization and iteration pitfalls.

Table 3: Comparison of iteration number between Simplex and QSM method [19]

Problem no.	Number of constraints	Number of variable	Iteration number	
			Simplex method	QSM
1	3	2	3	1
2	3	8	4	1
3	5	12	5	1

In literature, it is found that, among the BLSA, ε-OSD and QSM methods, there are pros and cons. The three studies conducted by previous researchers are identified according to their modification, knowledge contribution from the augmented method and also the computational performance. For instance, the BLSA method only analyzes the computational performance of termination but not the initialization and iteration. The same goes to ε-OSD and QSM methods, whereby these two methods only analyze the computational performance of initialization and iteration but not the termination.

Based on Table 4, the BLSA method comes out with a method that improvises the computational performance of termination as stated 'applicable' in the termination's column. However, computational performance for initialization and termination are not in the scope of BLSA method, thus, 'not applicable' is stated. As for ε-OSD algorithm and QSM method, both are in the same scope of computational performance which improvise the initialization and iteration of Simplex algorithm. However, the computational performance of termination is not in the scope of these two methods as this study is to combine these three methods to generate new methodologies to complete these pitfalls.

Table 4 The comparison of selected augmentation works of Simplex algorithm studies

Researcher	Method name	Modification	Knowledge contribution	Computation Performance		
				Initialization	Iteration	Termination
Zhu et. al, 2010 [4]	Basic Line Search Algorith m	Instead of moving from 0-D face (vertex) to an improved one, it move from1-D face (edge) to an improved one by pivot operation.	Skip some adjacent vertices and converges to optimal solution faster.	Not applicable	Not applicable	Applicable
Luh & Tsaih, 2002 [18]	ε-Optima lity Search Direction	Start an initial basic feasible solution near the optimal point. Release a corner point of feasible region within few iterative steps.	Helps for commencing better initial basic feasible solution. Consistently reduce number of required iterations.	Applicable	Applicable	Not applicable
Vaidya & Kasturiwale , 2016 [19]	Quick Simplex Method	Replace more than one basic variable simultaneously.	Involves less or at most equal number of iteration than in Simplex method.	Applicable	Applicable	Not applicable

3.0 METHODOLOGY

The methodology of this study started with the review of literature regarding the augmentation studies of Simplex algorithm. Literature review was conducted mainly to obtain the research gap within the current augmented-algorithm studies in order to dig out the scarce area. Then, once the gap is identified, the methodology will proceed to develop a theoretical background of the foundation to generate the prototype of new augmented-algorithms. Next, as the prototypes were developed, experiments were performed using mathematical computation. The results from the experiment were then used to verify the new augmented-algorithms. The verification was mainly to check whether the new augmented-algorithms worked accordingly and produced solution with LP problems which have objective function and constraint in a linear form.

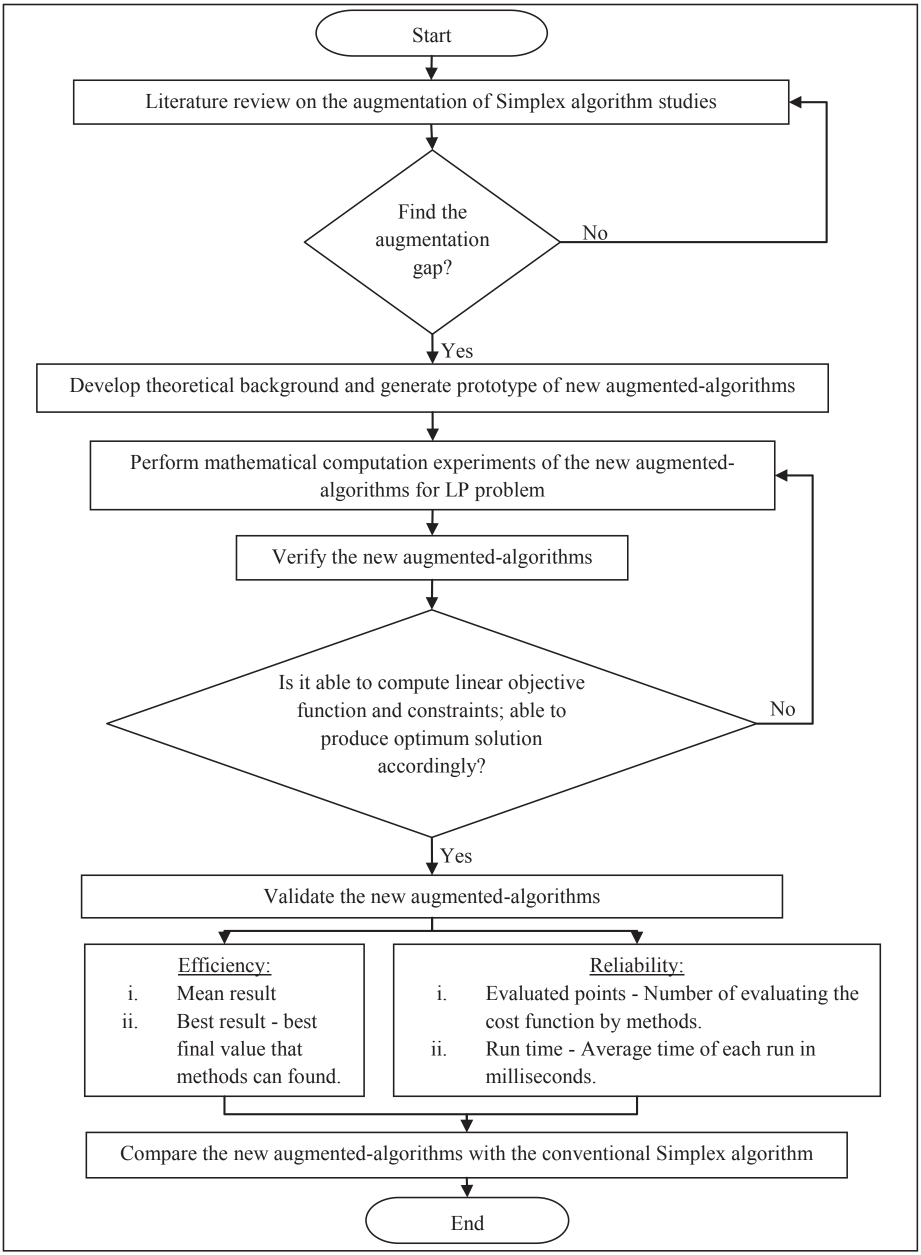

Figure 6: Process flow of the methodology of study

Later, verification was demonstrated by optimization procedures through diagram indicator of reliability and efficiency. The reliability of the new augmented-algorithms was tested through two indices which were “mean result” - the average value of all 1000 final results and “best result” - the best final value that methods can find. In addition, the efficiency of the new augmented-algorithms was tested through two indices which were “evaluated points” - the number of evaluating the cost function by methods and “run time”- the average time of each run in milliseconds. These, then, would be compared numerically using the conventional Simplex algorithm. The process flow of the methodology of this study can be referred in Figure 6.

4.0 RESULTS

Based on the literature review, two new methodologies are generated to enhance the computational performance in the Simplex algorithm as shown in Figure 7 and 8. The proposed methodologies namely Methodology 1 and Methodology 2 are based on the combination of methods between ε-OSD with BLSA and QSM with BLSA respectively. In the Methodology 1, the process flow starts with the mathematical formulation using LP’s model. The algorithm then proceeds by converting the problem into initial tableau as used in the conventional Simplex algorithm. Next, this is where the ε-OSD method is inserted in the process flow, as marked by the ‘red arrow’. The initialization and iteration algorithm are applying the ε-OSD method whereas the termination algorithm is applying the BLSA method. As the algorithm pivots, the process flow reaches the end once the feasible solution is obtained.

The same process applies to the Methodology 2 whereby the process flow starts with the mathematical formulation using LP’s model. The algorithm then proceeds by converting the problem into initial tableau as used in the conventional Simplex algorithm. The negative numbers in the Z-row is checked first before proceeding to the QSM method. Next, this is where the QSM method is inserted in the process flow, as marked by the ‘red arrow’. The initialization and iteration algorithm are applying the QSM method whereas the termination algorithm is applying the BLSA method. As the algorithm pivots, the process flow reaches the end once the feasible solution is obtained. Further explanation on how the algorithm works for Methodology 1 and Methodology 2 is summarized in Table 5 and compared with the conventional Simplex algorithm.

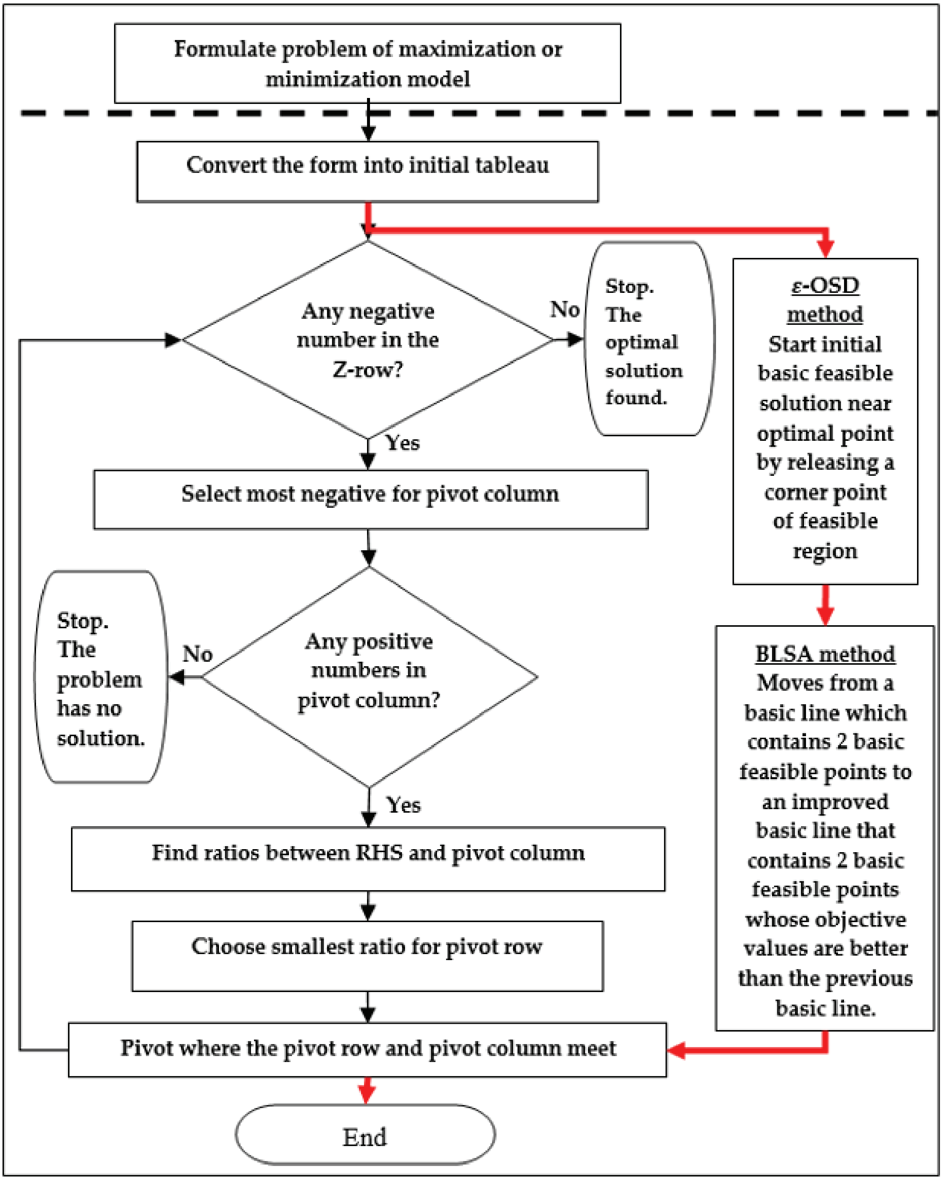

Figure 7: Methodology 1 (Combination of BLSA and ε-OSD methods)

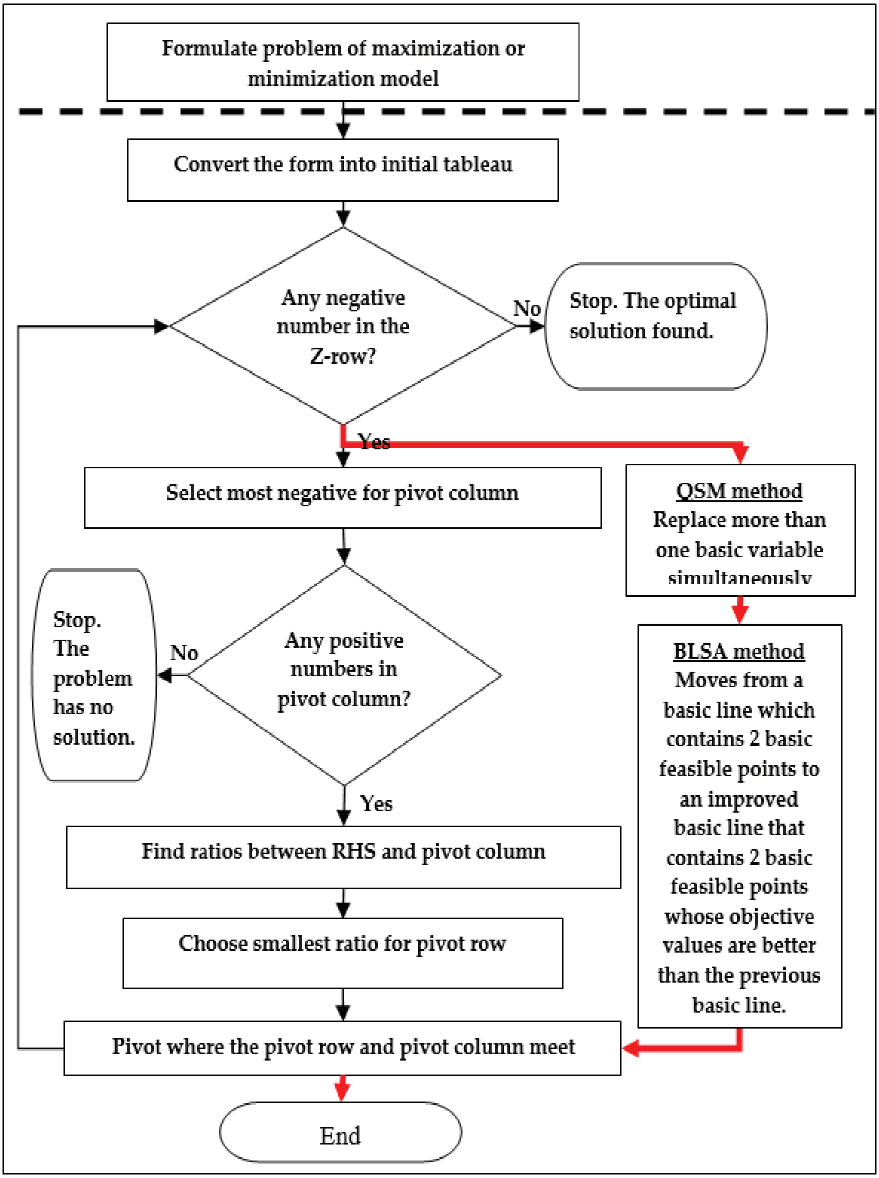

Figure 8: Methodology 2 (Combination of BLSA and QSM methods)

Table 5: Detail description flow of Methodology 1 and Methodology 2

<table>
<tr><th colspan="2">Method</th><th>New augmentation algorithm 1
BLSA +ε-OSD</th><th></th><th>New augmentation algorithm 2
BLSA +QSM</th><th></th></tr>
<tr><td rowspan="3">Computation performance</td><td>Initialization</td><td>A better initial corner point of feasible solution is released:
• Point moves consecutively to a better solution on the other boundary of polyhedron which is formed by the feasible region.</td><td rowspan="2">ε-OSD</td><td>For initial, more than one basic variable for entering and outgoing vector are introduced simultaneously.</td><td rowspan="2">QSM</td></tr>
<tr><td>Iteration</td><td>The process will be repeated till the point hits a corner point at the feasible region:
• The moving direction is a linear combination of the negative gradient direction of the objective function and a direction pointing towards the interior of the polyhedron.</td><td>The iteration continues by introducing simultaneously the multiple basic variable for entering and outgoing vectors until iteration reaches only one basic variable for entering and outgoing vector.</td></tr>
<tr><td>Termination</td><td>Using pivot operation, algorithm iterates from one feasible basis matrix to an improved feasible basis matrix till the optimal solution is found. Geometrically, in each iteration of the algorithm, the solution moves from the current line to an adjacent line that contains an edge of the constraint polyhedron.</td><td>BLSA</td><td>Using pivot operation, algorithm iterates from one feasible basis matrix to an improved feasible basis matrix till the optimal solution is found. Geometrically, in each iteration of the algorithm, the solution moves from the current line to an adjacent line that contains an edge of the constraint polyhedron.</td><td>BLSA</td></tr>
</table>

5.0 CONCLUSION

In conclusion, Simplex method is LP classical optimization method that is used as a constructive tool to educate and work out linear problem. It involves a step-by-step computation work, namely, initialization, iteration and termination. The studies on pitfalls from these three computational performance raise the augmentation of the Simplex method. Based on the augmentation studies, three methods are recognized which are BLSA (Basic Line Search Algorithm), ε -OSD (Efficient-Optimality Search Direction) and QSM (Quick Simplex Method). The combination of these three methods generates two new methodologies and are overcoming the pitfalls of the computational performance of the Simplex algorithm, striving toward its completion.

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